Physics.

Post Mid Sem Part

Lecture ①

Optics and Quantum Mechanics

Marks 40 + 10
end sem → 40, quizzes. (Best ②) → 10

→ EM wave is a solⁿ of Maxwell's Eqⁿ.

→ It is a wave formed by electric & magnetic field.

→ It is not a mechanical wave. ⇒ (No medium required to propagate!)

→ Light is nothing but EM wave.

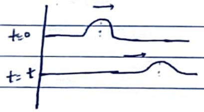
① → Any charge particle that is accelerating produces light.

→ accelerated charge particle produces EM waves.

② → Light also interacts with charge particles. eg. refraction, reflection. etc. Point charge particles can interact only through electric or magnetic field or both.

- Wave :-

There is transport of energy from one point to other without material displacement



this is wave propagation in time.

- longitudinal wave :- vibration of particles along the direction of propagation of wve.
  eg. sound wave.

- Transverse wave :- Vibration of particles ⊥ to the direction of propagation of wave.

* EM wave is a transverse wave.

---

* Electric field & Magnetic field, vibrates ⊥ to the direction of propagation.

How to describe wave and it's propagation mathematically

We are working in one dimensions
↳ one dimensional coordinate

- Wave travelling with speed v along x direction
  Assume that wave travels / propagates without any dissipation.

The pulse is described by the fⁿ $f(x)$ at $t=0$.
After a time t, a point on pulse has moved distance $\boxed{vt}$.

The profile $\psi(x,t)$
⇒ the height at x and time t, $\psi(x,t)$ is same as the height at $t=a$ and $x-vt$
↳ assuming wave is travelling in (+ve) x dirⁿ

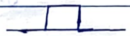
$$\left\{ \psi(x,t) = f(x, vt) \right\}$$

↳ Mathematical form of the waveform as a fⁿ of x & t.

eg. $f(x)\ e^{-\lambda x^2}$

$\psi(x,t) = e^{-\lambda(x-vt)^2}$ ✓✓

eg. $f(x) = A \quad -\frac{l}{2} < x < \frac{l}{2}$



0 otherwise

$\psi(x,t) = A \quad -\frac{l}{2} < (x-vt) < \frac{l}{2}$

$= 0$ otherwise.

- If the wave is travelling in -ve x dirⁿ.

$$\Rightarrow \underbrace{x}\longrightarrow \underbrace{x+vt}$$

★

- $\underbrace{t=\varepsilon}$ and waveform $f(x)$

At time t, the pulse has travelled "$x-v(t-\varepsilon)$"

$$\Rightarrow \psi(x,\vartheta) = f(x-v(t-\varepsilon))$$

~~$f(x,t)$~~

$$\boxed{\psi(x,\vartheta) = f(x-vt+\underbrace{\vartheta\varepsilon})}$$

initial phase.

independent of x & y

- We want to understand, what is the differential equation satisfied by the wave pulse?

★ Wave equation in 3 dimension

$$\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial}{\partial z^2}\right)\psi(x,y,z) = \frac{1}{v^2}\left(\frac{\partial^2\psi(x,y,z)}{\partial t^2}\right)\right]$$

- $\psi(x,y,z,t) = \psi(\vec{r},t)$

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$

- EM wave.

$$\left.\begin{aligned}\nabla^2\vec{E}(\vec{r},t) &= \frac{1}{v^2}\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2}\\ \nabla^2 B(\vec{r},t) &= \frac{1}{v^2}\frac{\partial^2 B(r,t)}{\partial t^2}\end{aligned}\right\}$$

★ Sinusoidal wave

1- dimension

$$f(x) = A\cos kx$$

or

$$A\sin kx$$

$$\begin{aligned}\psi(x,t) &= A\cos k(x-vt)\\ &= A\cos(kx-kvt)\\ &= A\cos(kx-\omega t)\end{aligned}$$

.... here $\boxed{kv=\omega}$

angular frequency

$$\boxed{\psi(x,t) = A\cos(kx-\omega t)}$$

plane wave in 3-dimension.

lecture②

Refer:

- Ajoy Ghatak, Optics:

㊀ wave propagation in one dim.

$$\psi(x,t) = f(x-vt+v\varepsilon)$$

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$$

$$\frac{\partial\psi(x,t)}{\partial x} = \frac{\partial f(x-vt+v\varepsilon)}{\partial x} = \frac{\partial f(z)}{\partial z}\cdot\frac{\partial z}{\partial z} = \frac{\partial f}{\partial z}$$

$$\boxed{z = x-vt+\varepsilon v}$$

$$\frac{\partial z}{\partial z} = 1 \quad \& \quad \boxed{\frac{\partial z}{\partial t} = -v}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2 f}{\partial z^2}\cancel{\frac{\partial z}{\partial z}}$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial f}{\partial z}\frac{\partial z}{\partial t} = -v\frac{\partial f}{\partial z}$$

* End sem que. → derive the result $\left\{ \frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \right\}$ for 3 dimensions.

$$\frac{\partial^2 \psi}{\partial t^2} = -v \frac{\partial^2 f}{\partial z^2} \frac{\partial z}{\partial t}$$

$$= v^2 \frac{\partial^2 f}{\partial z^2}$$

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Homework : What is the most general solⁿ to 1 dim wave eqⁿ?

End sem que. HW

$$\psi(x,t) = f(x - vt + \varepsilon v) + \tilde{f}(x + vt + \tilde{\varepsilon v})$$

$$\boxed{\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) = \left(\frac{\partial}{\partial x} - \frac{1}{v} \frac{\partial}{\partial t} \right) \left(\frac{\partial}{\partial x} + \frac{1}{v} \frac{\partial}{\partial t} \right)}$$

$$\left(\nabla^2 - \frac{1}{v} \frac{\partial^2}{\partial t^2} \right) \psi(\vec{r}, t) = 0$$

In 3D
1) Plane wave
2) Spherical wave
3) Cylindrical wave

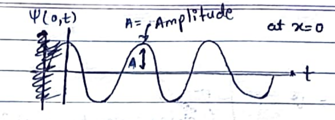
$\Psi = \psi(x, y, z, t)$

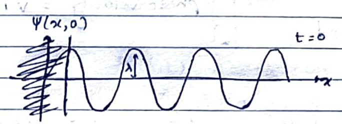
Harmonic wave
(Sinusoidal wave)

$f(x) = A \cos kx$
(or)
$A \sin kx$

$$\psi(x,t) = A \cos K(x - vt)$$
$$= A \cos(kx - \omega t) \quad \ldots \{\text{where } \omega = kv\}$$

phase → $\{\phi(x,t) = kx - \omega t\}$





$\omega T = 2\pi$

$\omega = \frac{2\pi}{T} = 2\pi\nu$ ← frequency

angular frequency.

$k\lambda = 2\pi$

$\left\{ k = \frac{2\pi}{\lambda} \right\}$

wave number → number of waves in an unit distance

wave length

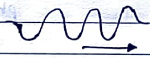
* Imp
$$\boxed{v = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda}$$

$$\boxed{v = \nu\lambda}$$

- The rate of change of ϕ w.r.t x

$\frac{\partial \phi}{\partial x} = k$

$\frac{\partial \phi}{\partial t} = -\omega$



Each point on the wave corresponding to certain value of phase.

The phase should remain constant if the points on the wave move Δx, there should be Δt, such that $\phi(x,t)$ remains constant.

$$\boxed{\frac{\partial \phi}{\partial x} \Delta x + \frac{\partial \phi}{\partial t} \Delta t = 0}$$

as $\phi(x,t) = (kx - \omega t)$

$k\Delta x - \omega \Delta t = 0$

$\frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v$.

phase speed

Hence,

$\{ \text{phase velocity} = v\hat{i} \}$

⊛ Exponential representation of trigonometric function.

Rectangular Coordinates | Polar variable

$Z = x + iy$

$\bar{Z} = x - iy$

$x = r\cos\theta$

$y = r\sin\theta$

$Z = r(\cos\theta + i\sin\theta)$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\{ \psi(x,t) = Ae^{i(kx - \omega t)} \}$$

$$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} \right) \psi(\vec{r},t) = 0$$

Amplitude "A"

phase ⇒ $\phi(\vec{r},t)$

wavefront at any given time t, the locus of all the points giving the same value of $\phi(\vec{r},t)$ constitute wavefront.

$$\psi(\vec{r},t) = A\cos(\vec{k}\vec{r} - \omega t)$$

Lecture 3

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = Re(e^{i\theta})$$

$$\sin\theta = Im(e^{i\theta})$$

$$\psi(x,t) = A\cos(kx - \omega t)$$

or

$$A\sin(kx - \omega t)$$

$$\psi_c(x,t) = Ae^{i(kx - \omega t)}$$

$$\psi = Re(\psi_c)$$

or

$$Im(\psi_c)$$

- $\cos\theta$, $\cos\theta'$

$z_1 = e^{i\theta}$, $z_2 = e^{i\theta'}$

$Re\, z_1 = \cos\theta$ $\quad$ $Re\, z_2 = \cos\theta'$

$$\boxed{Re\, z_1 + Re\, z_2 = Re(z_1 + z_2)}$$

$$\boxed{\frac{\partial Re(z_1)}{\partial x} = Re\left(\frac{\partial z_1}{\partial x}\right)} \checkmark$$

$$Re(z_1)\, Re(z_2) \neq Re(z_1 z_2)$$

$$\frac{Re(z_1)}{Re(z_2)} \neq Re\left(\frac{z_1}{z_2}\right)$$

Q) • $\cos\theta + \sin\theta'$

$\cos\theta + \cos\left(\frac{\pi}{2} - \theta'\right)$

to solve, we first converts all the trigonometric functions to one (single) trigo. function.

$$\left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)$$

- **Wavefront:**

It is the locus of all the points in phase at a given time.

eg. $\phi(x,t) = kx - \omega t$

$\phi(x, t_0) = C_0$

$x = \dfrac{C_0 + \omega t_0}{k}$

Things are interfering in higher dimension
1) 2 dim : wavefront is a curve.
2) 3 - dim : wavefront is a surface.

① Plane wavefront → plane wave
② Spherical wavefront → spherical wave
③ Cylindrical wavefront → cylindrical wave.

→ ① Plane Wavefront:

$$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,y,z,t) = 0$$

$$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$$

$$\psi(\vec{r},t) = A\cos(\vec{k}\cdot\vec{r} - \omega t)$$

$$\psi_c(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

here, c means complex notation.

$\vec{k}$ is arbitrary but constant velocity

* $\vec{k} = \hat{i}k_x + \hat{j}k_y + \hat{k}k_z$

provided

$$k^2 - \frac{\omega^2}{v^2} = 0$$

we know, $\boxed{k^2 = k_x^2 + k_y^2 + k_z^2}$

$$\phi(\vec{r},t) = \vec{k}\cdot\vec{r} - \omega t$$

$$\phi(\vec{r},t_0) = c_0$$

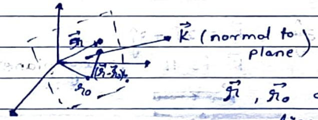
$$\vec{N} = \nabla(\phi(r,t_0) - c_0)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(\phi(r,t_0) - c_0)$$

$$= k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$$

$$= \vec{k}$$

→ This defines a surface for which the normal is everywhere constant vector $\vec{k}$



$\vec{r}$, $\vec{r}_0$ & $\vec{r} - \vec{r}_0$ vector lies within plane.

$t = 0$

$$\vec{k}\cdot\vec{r}_0 = c_0$$

$$\vec{k}\cdot\vec{r} = c_0$$

} subtract.

$$\vec{k}\cdot(\vec{r} - r_0) = 0$$

$\vec{k}\cdot\vec{R} = 0$ ⇒ solution is a plane.

$$\vec{R} = \vec{r} - \vec{r}_0$$

$$\{\vec{r} = \vec{R} + \vec{r}_0\}$$

$\vec{k}\cdot\vec{r} = C_0$ ⇒ $k(\hat{n}\cdot\hat{r}) = C_0$

$$\vec{k} = \hat{n}k$$

Follow in time

$\vec{k}\cdot\vec{r}\,' - \omega t = C_0$

$k(\hat{n}\cdot\vec{r}\,') - \omega t = C_0$
$\underbrace{\quad}$ ↳ +ive number

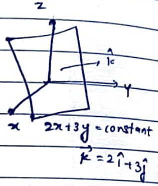
$\hat{n}\cdot\vec{r}\,'$ has to increase

⇒ the wavefront is moving in the direction $\hat{n}$

$\hat{n} = \hat{k}$

$kz - \omega t = C_0$

$\psi(\vec{r}, t) = A\cos(2x + 3y - \omega t)$



~~$\vec{k} = 2\hat{x} + 3\hat{y}$~~ $\vec{k} = 2\hat{i} + 3\hat{j}$

$\vec{k}$ is ∥ to x-y plane

~~$\vec{k}\cdot\vec{r} = 0$~~

$\hat{k}\cdot\vec{k} = 0$

unit vector. z axis ← $\hat{k}$; $\vec{k}$ → in xy plane.

$\vec{k}\cdot\Delta\vec{r} - \omega\Delta t = 0$

$k\,\Delta r_k - \omega\,\Delta t = 0$

⇒ the wave is moving along $\vec{k}$ with speed

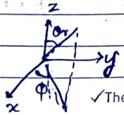
$\frac{\Delta r_k}{\Delta t} = \frac{\omega}{k}$

↳ phase velocity.

* Spherical Wavefront:-

Spherical polar coordinates.

(r, θ, ϕ)



$$\boxed{\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2}}$$

$\phi(\vec{r}, t) = \text{constant}$

↳ sphere

* ⇒ ϕ must be independent of θ & ϕ

$\psi(\vec{r}, t) = \psi(r, t)$

$$\underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}(r,t)\right)}_{\nabla^2\psi(r,t)} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\psi(r,t)) = 0$$

Homework

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2}}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\Rightarrow \frac{\partial^2}{\partial r^2}(r\psi) - \frac{1}{v^2}\frac{\partial^2(r\psi)}{\partial t^2} = 0$$

$r\psi = f(r - vt)$ ~~A~~

~~f cos~~ $f = A\cos(k(r \pm vt))$

$$\boxed{\psi(r,t) = \frac{A}{r}\cos(kr - \omega t)}$$ *

★ **Superposition Principle :-**

$$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$$

It is linear differential equation :

linear wherever ψ & it's derivatives appear, they appear with single power.

$\psi_1(\vec{r},t), \ldots, \psi_N(\vec{r},t)$

↳ N → no. of solutions to wave equation

$$\psi(r,t) = \sum_{i=1}^{N} C_i \psi_i(\vec{r},t)$$

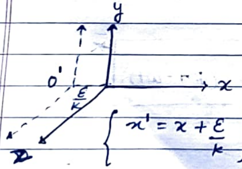
$\{ C_i \Rightarrow$ arbitrary constants $\}$

$$\boxed{\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = \sum_{i=1}^{N} C_i\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi_i(\vec{r},t) = 0}$$

eg)

$$\left.\begin{aligned}\psi_1 &= \psi_{01}\sin(kx-\omega t+\epsilon_1)\\ \psi_2 &= \psi_{02}\sin(kx-\omega t+\epsilon_2)\end{aligned}\right\}$$ both corroespond to same frequency

$\psi = A\sin(kx-\omega t+\epsilon)$ → there implicit assignment of the origin to some point



$\left\{ x' = x + \frac{\epsilon}{k} \right\}$ ★ Imp

$\psi = A\sin(kx'-\omega t)$

★ Note:- The phases are not important but the phase difference is important.

$\psi_1 = \psi_{01}\sin(kx-\omega t+\epsilon_1)$

$\psi_2 = \psi_{02}\sin(kx-\omega t+\epsilon_2)$

↳ $\psi_{02}\sin\left(k\left(x'-\frac{\epsilon_1}{k}\right)-\omega t+\epsilon_2\right)$ .... redefining w.r.t x'

$$\left\{\psi_2 = \psi_{02}\sin(kx'-\omega t+\underbrace{(\epsilon_2-\epsilon_1)}))\right\} ★$$

here, "c" means complex

$\psi_c = \psi_{1c} + \psi_{2c}$

$\psi_c = \psi_{01}e^{i(kx-\omega t+\epsilon_1)} + \psi_{02}e^{i(kx-\omega t+\epsilon_2)}$

$\psi_c = e^{i(kx-\omega t)}\left(\psi_{01}e^{i\epsilon_1} + \psi_{02}e^{i\epsilon_2}\right)$

$\psi_c = Me^{i(kx-\omega t)} = |M|e^{i(kx-\omega t+\theta)}$

$M = \psi_1 e^{i\epsilon_1} + \psi_2 e^{i\epsilon_2}$

$= |M|e^{i\theta}$

$|M| = \sqrt{(Re\ M)^2 + (Im\ M)^2}$

$|M| = \sqrt{(\psi_{01}\cos\epsilon_1 + \psi_{02}\cos\epsilon_2)^2 + (\psi_{01}\sin\epsilon_1 + \psi_{02}\sin\epsilon_2)^2}$

$$\boxed{|M| = \sqrt{\psi_{01}^2 + \psi_{02}^2 + 2\psi_1\psi_2\cos(\epsilon_1-\epsilon_2)}}$$

$\psi = Im\ \psi_c$

$= |M|\sin(kx-\omega t+\theta)$

→ harmonic wave of same frequency phase velocity

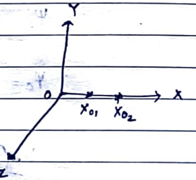
$\left\{ V_p = \frac{\omega}{k} \right\}$

$$|M|^2 = \psi_{01}^2 + \psi_{02}^2 + \underbrace{2\psi_{01}\psi_{02}\cos(\varepsilon_1 - \varepsilon_2)}_{\text{interference form.}}$$

- $\varepsilon_1 - \varepsilon_2 = 0, \pm 2\pi, \pm 4\pi, \ldots\ldots$
  $|M|^2 = (\psi_{01}^2 + \psi_{02}^2)$ ★

- $\varepsilon_1 - \varepsilon_2 = \pm\pi, \pm 3\pi, \ldots\ldots$
  $|M|^2 = (\psi_{01} - \psi'_{02})^2$ ★

$$\{\phi_i = kx - \omega t + \varepsilon_i\}$$



$$\varepsilon_2 - \varepsilon_1 = \phi_2 - \phi_1$$
$$= \frac{2\pi}{\lambda}(x_{02} - x_{01})$$

$+(\delta_2 - \delta_1)$ ← ~~this~~ this quantity may or may not be time dependent

phase difference due to source

$$\boxed{\varepsilon_2 - \varepsilon_1 = \frac{2\pi}{\lambda}(x_{02} - x_{01}) + (\delta_2 - \delta_1)}$$

∴ If $\delta_1 - \delta_2$ is independent of time, then the two sources are said to be coherent.

→ The two sources are coherent only of they produce interference patter.

[★] Waves having different frequency.

$$\psi_1 = A\cos((k + \Delta k)x - (\omega + \Delta\omega)t)$$

$$\psi_2 = A\cos((k - \Delta k)x - (\omega - \Delta\omega)t)$$

$$\psi = \psi_1 + \psi_2$$

$$\psi_c = \psi_{1c} + \psi_{2c} = Ae^{i((k+\Delta k)x - (\omega + \Delta\omega)t)} + Ae^{i((k+\Delta k)x + (\omega - \Delta\omega)t)}$$

$$\psi_c = Ae^{i(kx - \omega t)}\left[e^{i(\Delta kx - \Delta\omega t)} + e^{-i(\Delta kx - \Delta\omega t)}\right]$$

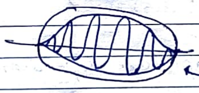
$$\psi_c = 2A\cos(\Delta kx - \Delta\omega t)\, e^{i(kx - \omega t)}$$

$\Delta\omega \to$ modulation frequency
$\Delta k \Rightarrow$ modulation wave number } of propagation.

$\Delta\omega < 1$

$$\psi = \mathrm{Re}\,\psi_c = \underbrace{2A\cos(\Delta kx - \Delta\omega t)}\cos(kx - \omega t)$$

we have enveloped region due to the variation of amplitude.



← wave packet is moving with velocity

$$V_g = \frac{\Delta\omega}{\Delta k}$$

↳ group velocity

$$V_p = \frac{\omega}{k}$$

is there a relation betn them?

Note: temporal pulse travels with the group velocity given by

Imp * $\left\{ V_g = \frac{1}{(dk/d\omega)} \right\}$

both waves indivisual velocities are equal.

V_{ψ_1} V_{ψ_2}

★ ~~Superposition~~

Imp

$$\boxed{\frac{\omega + \Delta\omega}{k + \Delta k} = \frac{\omega - \Delta\omega}{k - \Delta k}}$$

$$\frac{\omega + \Delta\omega}{\omega - \Delta\omega} = \frac{k + \Delta k}{k - \Delta k}$$

$$\frac{2\omega}{\omega - \Delta\omega} = \frac{2k}{k - \Delta k}$$

$$\frac{\omega}{k} = \frac{\omega - \Delta\omega}{k - \Delta k}$$

$$\omega k - \omega\Delta k = k\omega - \Delta\omega k$$

$$\boxed{\frac{\omega}{k} = \frac{\Delta\omega}{\Delta k}}$$

$\boxed{\text{group velocity} = \text{wave velocity}}$

$\boxed{\omega = k v}$

$\boxed{\omega = k . v}$

We want to create wave packet which is localised in space & time.

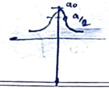
* Superposition of infinite no. of waves:

$$\psi(x,t) = \int_{-\infty}^{\infty} dk\, a(k)\, e^{i(kx - \omega(k)t)}$$

$\to$ amplitude dependent on k.

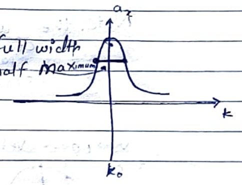
freq $\omega(k) \to$ dispersion relation.

* $\boxed{\omega(k) = kv}$



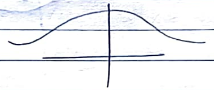
$\frac{1}{\tau_0}$

$$a(k) = a_0 e^{-\frac{(k-k_0)^2 \tau_0^2}{2}}$$

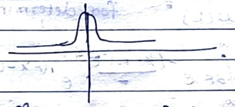


To find value of k for full width half maximum $= \frac{a_0}{2}$

$\tau\downarrow\downarrow \Rightarrow$ broadness increases



$\tau\uparrow\uparrow \Rightarrow$ broadness decreases



$$\psi(x,t) = \int_{-\infty}^{\infty} dk\, e^{-\frac{(k-k_0)^2 \tau_0^2}{2}} e^{i(kx - kvt)}$$

$$= \int_{-\infty}^{\infty} dk\, e^{-\frac{(k-k_0)^2 \tau_0^2}{2}} . e^{i((k-k_0)x - v(k-k_0)t)} \times e^{i(k_0 x - v k_0 t)}$$

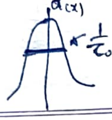
this can be taken out of integral as independent of k.

$$\psi(x,t) = e^{k_0(x - vt)} \int_{-\infty}^{\infty} dk\, e^{-\frac{k^2\tau_0^2}{2}} e^{i(kx - kvt)}$$

Imp result.

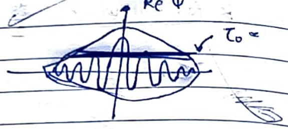
$$\boxed{= \frac{\sqrt{2\pi}}{\tau_0} e^{i(k_0 x - k_0 v t)} e^{-\frac{(x-vt)^2}{2\tau_0^2}}}$$

$$\boxed{\int_{-\infty}^{\infty} dx\, e^{-ax^2 + x(b+ic)} = \frac{\sqrt{\pi}}{\sqrt{a}} e^{-\frac{(b+ic)^2}{4a}}}$$

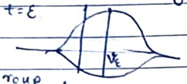
but 

$\propto \frac{1}{\tau_0}$ { amplitude $\propto \frac{1}{\tau_0}$ }

Note: the width of wave packet at (full width half maxima) is proportional to τ_0



v → phase velocity



group velocity = v

★ For general ω(k) ← dispersion relation for determining V_p & V_g relationship.

$$\psi(x,t) = a_0 \int_{-\infty}^{\infty} dk\, e^{-\frac{(k-k_0)^2 \tau_0^2}{2}} e^{i(kx-\omega(k)t)}$$

By Taylor's series → $\omega(k) = \omega(k_0) + (k-k_0)\left.\frac{d\omega}{dk}\right|_{k_0} + \dots$ ↳ ignoring

$$\omega(k) = a_0\int_{-\infty}^{\infty} dk\, e^{-\frac{(k-k_0)^2}{2}\tau_0^2} e^{i(kx-\omega(k_0)t-(k-k_0)\left.\frac{d\omega}{dk}\right|_{k_0} t)}$$

learn. Schrodinger eqn

$$\boxed{= a_0 e^{i(k_0x-\omega(k_0)t)} e^{-\frac{\left(x-\left.\frac{d\omega}{dk}\right|_{k_0} t\right)^2}{2\tau_0^2}}}$$

$$\boxed{V_p = \frac{\omega(k_0)}{k_0}} \qquad \boxed{V_g = \left.\frac{d\omega}{dk}\right|_{k_0}} ★$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = \hbar\frac{\partial\psi}{\partial t}$$

$$\omega(k) \propto k^2$$

$V_p = \frac{\omega_0}{k_0} \propto k_0$

$V_g \propto 2k_0$

∴ V_g will be greater than V_p in this case.

Self Notes }

λ = 5890 Å
= 5896 Å } Sodium lamp

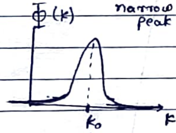
Δω ~ GHz
~ 10^9 Hz

★ Group Velocity → ω(k)

↳ velocity of a packet constructed by superposition of waves.

$$\psi(x,t) = \int dk\, \phi(k)\, e^{i\underbrace{(kx-\omega(k)t)}_{\psi(k)}}$$

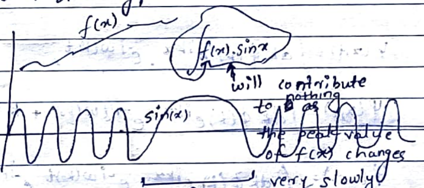
↑ amplitude of each wave; phase



only at k₀ frequency there is a bump → elsewhere $\phi(k) = 0$

• Principle of stationary phase

visualisation



∫f(x).sinx will contribute nothing to as the peak value of f(x) changes very slowly

at some point this is broader

here instead of using sin function, we are using exponential function.

since only for k = k₀ the integral has a chance to be non-zero

• Need that the phase becomes stationary w.r.t k at k₀.

↳ The slower it changes, it's better for integral ~~at~~ k₀

↳

$$\psi(k) = kx - \omega(k)t$$

$$\left.\frac{d\psi(k)}{dk}\right|_{k_0} = \left(x - \left.\frac{d\omega(k)}{dk}\right|_{k_0} t\right) = 0$$

at a stationary point stationary phase

↳ the "hump" in the packet will behave satisfying (the relation)

$$\therefore \boxed{x = \frac{d\omega}{dk}\Big|_{k_0} t}$$

$$* \therefore \boxed{\frac{d\omega}{dk}\Big|_{k_0} = V_{\text{group velocity}}}$$

$$\psi(x,t) = \int dk\, \phi(k)\, e^{i(kx-\omega(k)t)}$$

from taylor's expansion.

$$\omega(k) = \omega(k_0) + \underbrace{(k-k_0)\frac{d\omega}{dk}\Big|_{k_0}}_{} + O\left((k-k_0)^2\right) + \ldots$$

only these terms contribute & other are negligible.

$$\psi(x,t) = \int dk\, \phi(k)\, e^{ikx} \cdot e^{-i\omega(k)t}$$

substitute $\omega(k)$

$$\psi(x,t) = \int dk\, \phi(k) e^{ikx} \cdot e^{-i\left(\left(\omega(k_0) + k\frac{d\omega}{dk}\big|_{k_0} - k_0\frac{d\omega}{dk}\big|_{k_0} + \text{negligible}\right)t\right)}$$

$$\psi(x,t) = \int dk\, \Phi(k)\, e^{ikx} \cdot e^{-i\omega(k_0)t}\, e^{-ik\frac{d\omega}{dk}\big|_{k_0} t} \cdot e^{+ik_0\frac{d\omega}{dk}\big|_{k_0} t} \cdot e^{\text{negligible}}$$

1

~~$\psi(x,t) = \int dk\, \Phi(k)\, e^{ikx}$~~

$$\psi(x,t) = e^{-i\omega(k_0)t} \cdot e^{ik_0\frac{d\omega}{dk}\big|_{k_0} t} \int dk\, \Phi k\, e^{ik\left(x - \frac{d\omega}{dk}\big|_{k_0} t\right)} \quad *$$

Condition ①

$$\psi(x,t,0) = \int dk\, \Phi k\, e^{i(kx)}$$

in this term we have replaced $\underbrace{x \text{ form } \psi(x,t,0)}$ to $\left(x - \frac{\partial\omega}{\partial k}\big|_{k_0} t\right)$ *

some ~~x'~~ ~~lets~~ say.

Condition ② *

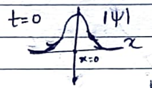
$\psi(x,t)$ if its complex no., it is hard to see the bump because it may be in imaginary part & not in real part or real part & not in imaginary part so take modulus of $\psi(x,t)$

$|\psi(x,t)|$ ← magnitude

✓ ☒

From condition ① & ②

$$|\psi(x,t)| = \left|\psi\left(x - \frac{d\omega}{dk}\Big|_{k_0} t, 0\right)\right|$$



→ ↓ this will have it's peak when $x - \frac{d\omega}{dk}\big|_{k_0} t = 0$

Huygen's Principle: ⑫ + Probems.

Newton ↳ corpuscular Theory: reflection, refraction, propagation of light in Vaccum

- Huygens wanted to explain all these phenomenon using the wave nature of light.

↳ Huygen's theory was a geometrical construction of propagation of wavefront.

statement:

* Given a wavefront, then every pt. of the wavefront acts as a source of the secondary spherical wavelets ~~wavefront~~. The wavefront is the envelope of all these waves is a spherical wave.

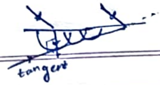
* If the frequency & speed of propogation are ν & v respectively then the secondary wavelets have same frequency ν and same speed v.

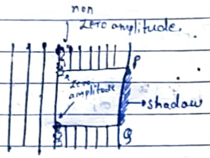
next wavefront at t + Δt

consider spherical wave with radius $v\Delta t$ distance) ~~path~~ spherical

Secondary & primary source),

in between two or more wavefront, draw a mutual tangent.

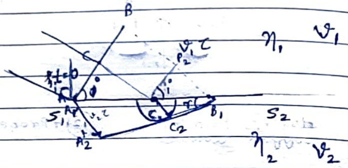




Huygens proposed that not every point on the spherical wavelet, we have non zero amplitude

- Interference

Snell's Law derivation



i → angle of incidence

r → angle of reflection

Note: when light travels from optically rarer to optically denser medium, the frequency of light remains same but the velocity & wavelength of light changes.

$BB_1' = v_1 \tau$

$CC_1' = v_1 \tau_1$

$AA_2 = v_2 \tau$

$C_1C_2 = v_2(\tau - \tau_1)$

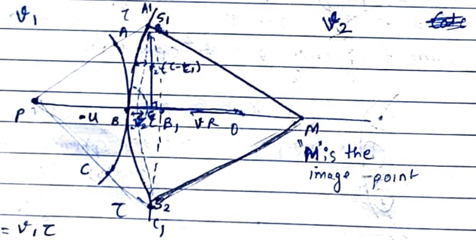
$$\frac{\sin i}{\sin r} = \frac{\left(\frac{B_1B_2}{C_1B_1}\right)}{\left(\frac{C_1C_2}{C_1B_1}\right)} = \frac{B_1B_2}{C_1C_2}$$

$$\frac{\sin i}{\sin r} = \frac{v_1(\tau-\tau_1)}{v_2(\tau-\tau_1)}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\star \boxed{n_1 \sin i = n_2 \sin r}$$

* Len's Law derivation.



$AA_1 = v_1 \tau$

$CC_1 = v_1 \tau$

$\cos x = 1 - \frac{x^2}{2}$ ~~lim 0~~

Calculate the length AG using three spherical surfaces

$$(AG)^2 = (A_1O)^2 - (OG)^2$$
$$= (R)^2 - (BO - BG)^2$$
$$= R^2 - (R - BG)^2$$
$$= BG\,(2R - BG)$$

$|u|$ & $v >> 1$

$R > 1$

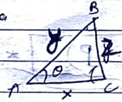
- Similarly do it for two other spherical surfaces.

$$\left\{ \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \right\}$$

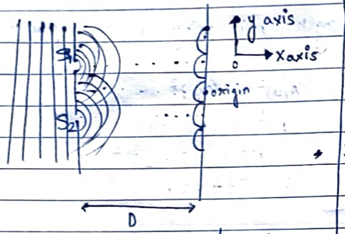
$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$ lens formula $\Big/$ $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

- cerenkov radiation

- cos formula



$$\cancel{x^2} \left\{ \cos\theta = \frac{x^2 + y^2 - z^2}{2xy} \right\}$$



S_1: coordinates → $(0, \frac{d}{2}, -D)$

S_2: coordinates → $(0, -\frac{d}{2}, -D)$

P $(x,y,0)$

S_1: $\vec{E_1} = \vec{E_{01}} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)$ ★

~~$= \vec{E_{01}} \cos($~~ $= \vec{E_{01}} e^{i\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)}$

★ S_2: $\vec{E_2} = \vec{E_{02}} e^{i\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)}$

$E_1 \parallel E_2$

$$\boxed{\Psi \sim \frac{A}{r}}$$

On composing the amplitudes of S_1P & S_2P with each other they are almost equal but when the comparisions of S_1P & S_2P is w.r.t the wavelength then they differ.

$$\vec{E} = \vec{E_1} + \vec{E_2} = E_{01} e^{i\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)} + E_{02} e^{i\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)}$$

$$\vec{E} = e^{i\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)} \left(\vec{E_{01}} + \vec{E_{02}} e^{i\delta}\right)$$

$$\boxed{\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)} \star$$

$I = k|E^2|$

$$|E|^2 = \vec{E} \cdot \vec{E}^*$$

$$= k\left(\vec{E_{01}} + \vec{E_{02}} e^{i\delta}\right)\left(\vec{E_{01}} + \vec{E_{02}} e^{-i\delta}\right)$$

$$= k\left(|E_0|^2 + |E_{02}|^2 + 2|E_{01}||E_{02}|\cos\delta\right)$$

$$= 2k|E_{01}|^2(1 + \cos\delta)$$

$$\therefore \boxed{I = 4k|E_0|^2 \cos^2\frac{\delta}{2}} \star$$

Max : intensity $\quad \delta = 2n\pi$; ~~2π~~ .... $n = 0, 1, ...$

$$S_2P - S_1P = n\lambda$$

$$\boxed{I = 4k|E_0|^2}$$

Min: Intensity

$$\delta = (2n+1)\pi$$

$$S_2P - S_1P = \frac{(2n+1)\lambda}{2}$$

★ $S_2P - S_1P = \Delta$

locus of all the points on the screen having same path difference Δ

$$S_2P = \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + D^2}$$

$$S_1P = \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2}$$

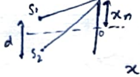
$$\sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + D^2} - \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2} = \Delta$$

$$\left(\sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + D^2}\right)^2 = \left(\Delta + \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2}\right)$$

squaring on both sides

$$\left(y + \frac{d}{2}\right)^2 = \Delta^2 + 2\Delta\sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2} + \left(y - \frac{d}{2}\right)^2$$

$$2yd = \Delta^2 + 2\Delta\sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2}$$



$$\left\{ \Delta x \times \frac{2\pi}{\lambda} = \Delta\phi \right\}$$

$$x_n = \frac{\Delta D}{d} \Rightarrow \left\{ \Delta = \frac{d x_n}{D} \right\}$$

$$(2yd - \Delta^2)^2 = 4\Delta^2\left(x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right)$$

$$4y^2d^2 + \Delta^4 - \underline{4yd\Delta^2} = \underline{4\Delta^2}\left(x^2 + y^2 + \frac{d^2}{4} - \underline{yd} + D^2\right)$$

$$\boxed{y^2(d^2 - \Delta^2) - x^2(\Delta^2) = \Delta^2\left(d^2 - \frac{\Delta^2}{4}\right) + \Delta^2 D^2}$$

→ equation of hyperbola.

Note:- If we will also consider point in the given space having z coordinate too, then the pattern will be formed over the surface of paraboloid.

y = constant. line is ⊥ to the x-y plane

y is a constant straight line near origin $x \to 0$

$$\boxed{y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\sqrt{(x^2 + D^2) + \left(d^2 - \frac{\Delta^2}{4}\right)}}$$

after some approximations

Fringe width

$$\beta = y_{n+1} - y_n$$

$$y_{n+1} = \frac{\Delta_{n+1}}{\sqrt{d^2 - \Delta_{n+1}^2}} D$$ ~~considering~~

$$y_n = \frac{\Delta_n}{\sqrt{d^2 - \Delta_n^2}} D$$

$$\Delta_n = n\lambda$$

$$\left\{ B = \left(\frac{\Delta_{n+1} - \Delta_n}{d}\right) D = \frac{\lambda D}{d} \right\}$$

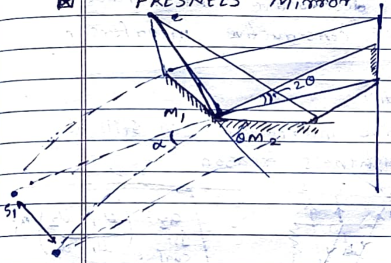
Q $D = 50\text{ cm}$, $d = 0.02\text{ cm}$, $y = 0.5\text{ cm}$

$S_1P = 50.0024\text{ cm}$
$S_2P = 50.0026\text{ cm}$ } Values that are obtained from, $\lambda = 6000\,\text{Å}$

$$S_2P = \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + D^2}$$

$$S_1P = \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + D^2}$$

☒ FRESNEL'S Mirror

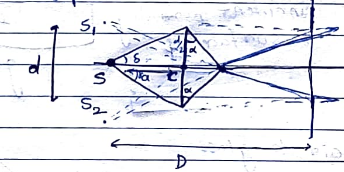


$$\beta = \frac{\lambda D}{\tilde{d}}$$

$$\tilde{d} = \alpha R$$

$$\boxed{\alpha = 2\theta}$$

- Fresnel Biprism:- Face of ~~fun~~ Brism is put ∥ to the surface).



$$\tan\delta = \frac{d/2}{a}$$

$$d = 2a\tan\delta$$

$\alpha \sim 0.5°$, μ is refractive index of prism

$$\left. \beta = \frac{\lambda D}{d} \right\}$$

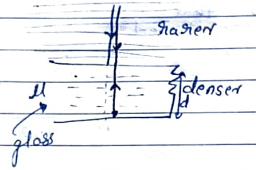
$$d = 2a(\mu - 1)\alpha$$

$$\frac{2\pi(\mu-1)d}{\lambda}=\phi$$

* Interference pattern :-

using division of amplitude:-

Eye (uniform illumination

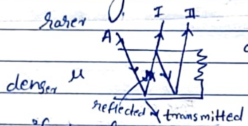


we have two reflected beams, one from top surface and other from bottom surface.

$$2\mu d \begin{cases} = m\lambda & \text{destructive interference} \\ = (m+\frac{1}{2})\lambda & \text{constructive interference} \end{cases}$$

uniform illumination

destructive interference will still have some brightness reason



as

$$\boxed{a_r = \left(\frac{\mu_1-\mu_2}{\mu_1+\mu_2}\right)a_i}$$ 1st surface,

$$\boxed{a_t = \left(\frac{2\mu_1}{\mu_1+\mu_2}\right)a_2}$$

if the lines incident normally to surface

generalise

$\mu_1 < \mu_2$ air to glass

$\mu_1 = 1$, $\mu_2 = 1.5$ for glass & air.

$$\boxed{a_r = -\frac{0.5}{2.5}a_i = -\frac{1}{5}a_i}$$

~~generalised~~ ~~formula~~

$$\boxed{\frac{a_r^2}{a_i^2} = 0.04}$$

---

for first reflection ✓

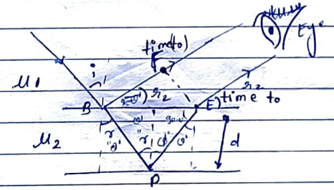
$$a_r = \frac{\mu_1-\mu_2\, a_i}{\mu_1+\mu_2}, \quad a_t = \frac{2\mu_1}{\mu_1+\mu_2}a_i$$

for ~~first~~ second reflection behaves as a_i

$$\begin{cases} a_r = \frac{(\mu_2-\mu_1)}{(\mu_1+\mu_2)}\left(\frac{2\mu_1}{\mu_1+\mu_2}\right)a_i \\ \tilde{a}_t = \left(\frac{2\mu_2}{\mu_1+\mu_2}\right)\left(\frac{\mu_2-\mu_1}{\mu_1+\mu_2}\right)\left(\frac{2\mu_1}{\mu_1+\mu_2}\right)a_i \end{cases}$$

$\mu_1 = 1$, $\mu_2 = 1.5$

$$\frac{\tilde{a}_t^2}{\tilde{a}_i^2} = 0.036.$$



uniform ~~beam~~ illumination

$\mu_1 \sin(i) = \mu_2 \sin(r)$

here $\boxed{r=\theta'}$

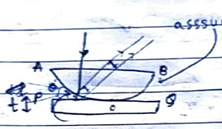
$BD = \frac{d}{\cos\theta'}$

$BF = \frac{d}{\cos\theta'}$

$\Delta = 2\mu_2 BD - \mu_1 BF$
$= 2\mu_2 d\cos\theta'$

~~$\mu_1 \sin(r_2) = \mu_2 \sin(90-\theta')$~~
~~$\Rightarrow \mu_1 \sin(r_2) = \mu_2 \cos\theta'$~~

Q) Prove.



assume this to be almost flat $\Rightarrow \cos\theta' =$

$R >> t$

very large radius of curvature

~~2μt cos~~

R is very large

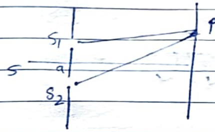
$2\mu t \cos\theta' \approx 2\mu t$

$2\mu t = m\lambda$ destructive

$= (m+\frac{1}{2})\lambda$ constructive

Homework :- include the refracted beam to this analysis
consider three rays and then solve

31/05/23



Stationary Interference

$I \propto \cos^2\left(\frac{\delta}{2}\right) = \frac{1}{2}(\cos\delta + 1)$

consider the situation when δ is function of time

$\bar{I} \propto \frac{1}{2}(1 + \overline{\cos\delta})$

$\bar{f(t)} \propto \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} f(t)\,dt$

time resolution of a human eye ≈ 0.1 sec

Now, if δ is random
cos(δ) is random
↳ is fluctuating in time τ
randomly betⁿ -1 to +1.

We will not have stationary interference

Average value

$\therefore \bar{I} \propto \frac{1}{2}(1 + \overset{0}{\cancel{\overline{\cos\delta}}})$

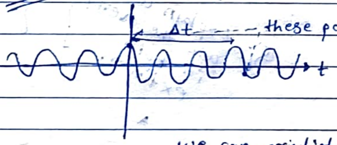
$\therefore \bar{I} \propto \frac{1}{2}$

Sources are not coherent

Q) Why δ is not constant in time?
⇒ due to the source of emmision of light.

$x = x_0$

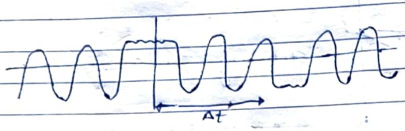
$E = E_0 \cos(\omega t)$



these points (any two points are correlated)

we can predict the nature of sinusoidal wave

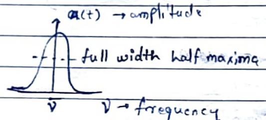
we can predict the phase at any point on the curve at any time.



In the above fig. we cannot predict the phase at $t > \Delta t$
∴ we can conclude that two particles at different x are not inter related.

- Atomic transition

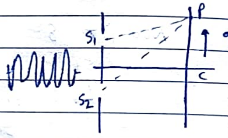
each of the wave packet will not be corelated.



coherence time or temporal coherence ↘

$$\tau_c = \frac{1}{\Delta\nu}$$

↗ determines how closer a wave pattern is towards a sinusoidal wave



as we move away from central line the intensity of overlap will decreases

$$\left.\begin{matrix} t - \frac{S_1P}{c} ✓ \\ t - \frac{S_2P}{c} ✓ \end{matrix}\right\}$$

$$\left(t - \frac{S_1P}{c}\right) - \left(t - \frac{S_2P}{c}\right) < \tau_c$$

$$\left\{= \frac{S_1P + S_2P}{c} < \tau_c\right\}$$

Q] Explain when will the overlap in both extinct.

★

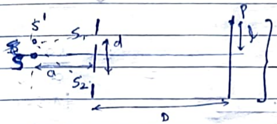
Imp Q)

- Neon lamp $\tau_c = 10^{-10}$ s
$l_c = 3$ cm

↓ coherent length.

$$\boxed{l_c = c\tau_c}$$

Cadmium lamp $\tau_c \sim 10^{-9}$ s
$l_c = 30$ cm



overlap of two interference pattern

Note S₁ & S₂ are equidistant from S.
but S₁ & S₂ will not be equidistant from S'

So the extra optical path difference of $(S'S_1 - S'S_2)$ is introduced.

- $(SS_1' - S'S_2) = \frac{\lambda}{2}$ at P will produce the interference which will destroy the interference pattern due to S.
↳ Brigt will be dark
& dark will be bright.

∴ $S'S_1 = \sqrt{a^2 + \left(\frac{d}{2} - l\right)} \approx a + \frac{1}{2a}\left(\frac{d}{2} - l\right)^2$
use binomial expansion.

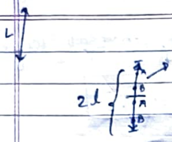
$$SS_2' \approx a + \frac{1}{2a}\left(\frac{d}{2} + l\right)^2$$

Now, $S'S_1 - SS_2' = \lambda/2$

$$\frac{1}{2a}\left(\frac{d}{2} + l\right)^2 - \frac{1}{2a}\left(\frac{d}{2} - l\right)^2 = \frac{\lambda}{2}$$

$$\frac{2dl}{2a} = \frac{\lambda}{2} \Rightarrow \boxed{l = \frac{a\lambda}{2d}} ★$$

$d^2 \cos 30° =$

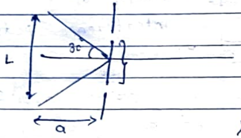


lateral length of source.

distance points the each L will ~~cancelee~~ cancel each other.

$$\star \quad \boxed{L = \frac{a\lambda}{d} = \text{spatial coherence length}}$$

Q} eg)



$$L < \frac{\lambda a}{d}$$ } to see interference pattern

$\lambda = 5000 Å$, what will be d so that we have interference pattern

$$L < \frac{2a}{d}$$

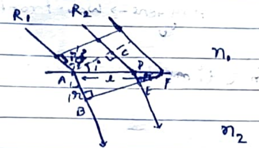
$$d < \frac{\lambda a}{L} = \frac{\lambda}{\theta} = 5000 \times 10^{-8}$$

$= 0.005 cm$

$$\theta = \frac{32\pi}{60 \times 180}$$

• Proof:- Refraction using Huygen's principle

Practice (Rough work)



R_1 travels AB dist in τ time
& R_2 travels CD + DE dist in τ time

$AB = l \sin(r)$ .... ①

~~$\sin(i)$~~ $(l - DF)\sin(i) = CD$ ... ②

$DF \sin(r) = DE$ --- ③

here $AB = v_2 \overset{\text{time}}{\tau}$

$CD = v_1 \tau_1$

$DE = v_2(\tau - \tau_1)$

} eq^n ① becomes $v_2 \tau = l \sin(r)$

eq^n ② becomes $(l - DF)\sin(i) = v_1 \tau_1$

eq^n ③ becomes $DF\sin(r) = v_2(\tau - \tau_1)$

~~Add eq~~

subtract eq^n ① & ③

$$(l - DF)\sin(r) = \cancel{(v_2\tau - v_2\tau)} + v_2\tau_1$$

$\therefore (l - DF)\sin(r) = v_2 \tau_1$ .... eq^n ④

divide eq^n ② & ④ we get

$$\frac{\sin(i)}{\sin(r)} = \frac{v_1 \tau_1}{v_2 \tau_2} = \frac{v_1}{v_2} = \frac{v_1 \times c}{v_2 \times c} = \left(\frac{c}{v_2}\right) \times \left(\frac{v_1}{c}\right)$$

$$\boxed{\frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1}}$$

as source has same frequency in every medium.

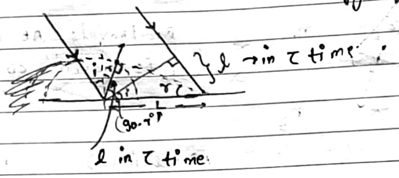
$$\frac{\sin(i)}{\sin(r)} = \frac{v_1}{v_2} = \frac{\lambda_1 f}{\lambda_2 f} = \frac{\lambda_1}{\lambda_2}$$

· Proof: Refection using Huygen's Principle.

Date:
Page No.:

Huygens → Wavefronts

*



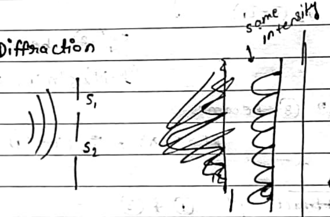
By ~~Ba~~ congruence rule

Both are right angled triangles with one side same and common hypotenuse

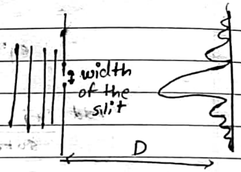
$\sin i = \frac{l}{L}$ | $\sin r = \frac{l}{L}$

∴ $\boxed{i = r}$ hence, proved.

☀ Diffraction



Let S_1 has some size



S_1 & S_2 are point sources.

here, on point source at each ~~[illegible]~~ S_1 & S_2

diffraction pattern

here, many point sources are present in the width of slit

* Fresnel diffraction (near field diffraction)

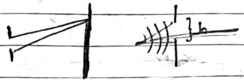
Source & Screen or at least one of them is very close to the diffracting slit

D ~ b

✓The Good Paper

Date:
Page No.

D~b



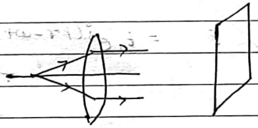
The amplitude at every point of the diffracting slit are different

The amp at a point P due to different points on the slit are different.

* Fraunhofer diffraction (far field diffraction) : Screen and the source both are at infinite distance.

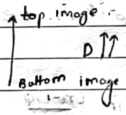
Fig 10.2

D >> b

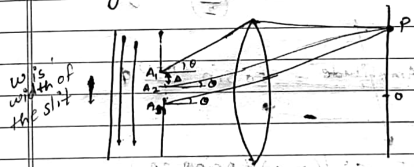


↳ Diagram bottom to top image

D ↑↑



* Single Slit Diffraction:-



Diffracting slit as a collection of many point sources.

we have point n point sources, $A_1, A_2 \ldots A_n$

$\boxed{b = (n-1)\Delta}$

Eventually we have to take

$\Delta \to 0$ & $n \to \infty$

$A_1 : \frac{C}{R} = \frac{C}{R}$ keeping b fixed.

$A_2 : \frac{C}{R+\Delta} = \frac{C}{R}$ as $D >> b$

✓The Good Paper

$t = \frac{r}{c}$, $t = \frac{r'}{c}$

the different distances travelled by the spherical wavefront gives optical path difference

$$= (\Delta \sin\theta)\frac{2\pi}{\lambda} = \phi$$

$$E = E_0 e^{i(kr-\omega t)} + E_0 e^{i(kr-\omega t-\phi)} + E_0 e^{i(kr-\omega t-2\phi)} + \dots + E e^{i(kr-\omega t-(n-1)\phi)}$$

as N points.

$$E = E_0 e^{i(kr-\omega t)}\left(1 + e^{-i\phi} + \dots + e^{-i(n-1)\phi}\right)$$

$$\left\{ \text{Amplitude} = E_0 \frac{\sin\frac{(n-1)\phi}{2}}{\sin\left(\frac{\phi}{2}\right)} \right\} \star$$

~~Amplitude = $E_0 \sin \frac{(n-1)}{2}\phi$~~

$$\text{Amp} = E_0 \frac{\sin\left((n-1)\left(\Delta\sin\theta\frac{2\pi}{\lambda}\right)\right)}{\sin\left(\frac{\Delta\pi}{\lambda}\sin\theta\right)}$$

$$\star \quad \boxed{\text{Amp} = E_0 \frac{\sin\left(\frac{b}{\lambda}\pi\sin\theta\right)}{\sin\left(\frac{\pi b}{n\lambda}\sin\theta\right)}}$$

$$\left\{ B = \frac{\pi b \sin\theta}{\lambda} \right\}$$

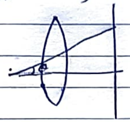
intensity

$I \propto A^2$

constant

$$I(\theta) = \frac{k E_0^2 \sin^2\left(\frac{\pi b}{\lambda}\sin\theta\right)}{\sin^2\left(\frac{\pi b}{n\lambda}\sin\theta\right)}$$

$$\left\{ \lim_{\theta\to 0} \sin\theta = \theta \right\}$$



$$I(0) = k E_0^2 \frac{\left(\frac{\pi^2 b^2}{\lambda^2}\right)}{\left(\frac{\pi^2 b^2}{n^2\lambda^2}\right)} = k E_0^2 n^2$$

$$\frac{I(\theta)}{I(0)} = \frac{\sin^2\left(\frac{\pi b \sin\theta}{\lambda}\right)}{n^2 \sin^2\left(\frac{\pi b}{n\lambda}\sin\theta\right)}$$

~~$\pi b \sin$~~

$n \to \infty$ keeping b fixed

$$\frac{I(\theta)}{I(0)} = \frac{\sin^2(B)}{B^2}.$$

$B = \frac{\pi b \sin\theta}{\lambda}$

⇓

~~$\sin^2$~~(.

Incomplete.