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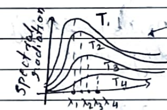
- Dawn of Quantum Theory :
  1. Black body
  2. Photoelectric effect.

When we heat an object, it emits radiation.
They also absorb. ⇒ objects are not only emitters
they are also absobers.

An ideal situation :- The perfect absorber when we heat the blackbody, it emits radiation, in all possible frequency

when we consider the cavity at finite temp T.
After some time, the radiation inside the cavity comes to equilibrium with it's surrounding.
The radiation in equilibrium with the cavity is called blackbody radiation

- leaking radiation:
  we are interested in understanding the property of leaking radiation.



Wien's displacement application here $T \propto \frac{1}{\lambda}$

$T_1 > T_2 > T_3 > T_4$
$\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

- Stefan - Boltzmann
  Blackbody at absolute temp T

$R \equiv$ energy emitted / time / area

Temp is in kelvin

$\boxed{R = \sigma T^4}$ ← Stefan's law

$\frac{\text{Power}}{\text{Area}}$

σ = stefan's constant
$= 5.67 \times 10^{-8}\ W/m^2 K^4$

Note: if power radiated is only asked then simply find $\boxed{\text{Power per } m^2 = \sigma T^4}$

if power for some area (A) is asked then $\boxed{P_{total} = \sigma T^4 A}$

Conversion : formulae $\boxed{°F = \left(°C \times \frac{9}{5}\right) + 32 = (°C \times 1.8) + 32}$ *

$\boxed{°K_{(kelvin)} = °C + 273.15}$ *

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• **Wien's displacement law**

λ_m is the wavelength of max^m energy

$\nu_{max} \propto T$

$\boxed{\nu_{max} = (5.8 \times 10^{10}\ Hz/K)(T)}$

Proportionality constant *

$\lambda_m T = \text{constant} = 3 \times 10^{-3}\ mK$

$\{\lambda_m T = 3 \times 10^{-3}\ mK\}$

• **Rayleigh - Jeans Formula:**

→ standing wave formation

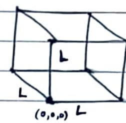
Time independent situation

Imp* $\boxed{\psi(\vec{r},t) = A \sin\left(\frac{n_1 \pi x}{L}\right) \sin\left(\frac{n_2 \pi y}{L}\right) \sin\left(\frac{n_3 \pi z}{L}\right) \cos(\omega t)}$

where n_1, n_2, n_3 are integers

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$$ Apply on this

* $$\left\{ n_1^2 + n_2^2 + n_3^2 = \frac{\omega^2 L^2}{c^2 \pi^2} \right\}$$ Result we get



For each combination (n_1, n_2, n_3) satisfying the eq^n give us a standing wave.

$$n_1^2 + n_2^2 + n_3^2 = \frac{4\pi^2 \nu^2 L^2}{\nu^2 \lambda^2 \pi^2} = \frac{4L^2}{\lambda^2}$$

$$\left\{ n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2} \right\}$$ * both must be solved simultaneously

n_1, n_2, n_3 = Integers $\Rightarrow \{ n_1, n_2, n_3 \geq 0 \}$

Distinct solution, linearly independent solutions

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"show that"

Endsem questions

* 1) Energy contained in each of these modes?
* 2) How many modes are there in each wavelength?

→ Each mode is an oscillator

Boltzmann law: The probability for an oscillator to have energy E is given by

$$\left\{ P(E) = \frac{e^{(-E/k_B T)}}{\int_0^\infty e^{-E/k_B T}\, dE} \right\}$$ *

→ Average Energy / Oscillator

$$\bar{E} = \frac{\text{Total energy}}{\text{Total oscillation}}$$

$N_0 \equiv$ Total oscillation.

Then the no. of oscillators having energy bet^n E and $E+dE$ is $N_0 P(E) dE$.

∴ Total energy

E $E+dE$

$$\left(\frac{(E+dE)+(E)}{2}\right) N_0 P(E) dE \approx \left(E + \frac{dE}{2}\right) N_0 P(E) dE$$

dE is very small → negligible

$$\approx E N_0 P(E) dE$$

$$\bar{E} = \int_0^\infty E\, P(E)\, dE = \frac{\int_0^\infty E e^{-E/k_B T} dE}{\int_0^\infty e^{-E/k_B T} dE} = k_B T$$

$\boxed{\bar{E} = k_B T}$

Average energy per oscillator is independent of frequency

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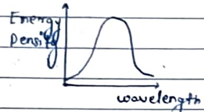


$(n_1, n_2, n_3) \longrightarrow$ +ve integers

1) Average energy of modes / oscillator $\boxed{\bar{E} = K_B T}$
2) Density of oscillator / modes for a given frequency

$(n_1, n_2, n_3) \longrightarrow$ +ve integer

$$\left\{ n_1^2 + n_2^2 + n_3^2 = \left(\frac{2L}{\lambda}\right)^2 \right\}$$

✓ now fix a λ and choose (n_1, n_2, n_3) to get a standing wave

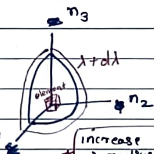
$$\left\{ \lambda = \frac{2L}{\sqrt{n_1^2 + n_2^2 + n_3^2}} \right\}$$

here fix (n_1, n_2, n_3) and then get wavelength of the standing wave.

equation of sphere

$$\left\{ x^2 + y^2 + z^2 = (r)^2 \right.$$

$$\boxed{\text{radius} = \frac{2L}{\lambda}}$$



Any point inside the sphere of radius $\frac{2L}{\lambda_0}$ will correspond to a standing wave of wavelength $> \lambda_0$.

★ increase in radius → decrease in wavelength
{ $dr \to +ve$
$d\lambda \to -ve$

In outside sphere also, we can get standing wave, but it's wavelength $< \lambda_0$

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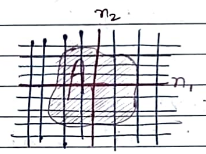
- Density of modes: Let $dN(\lambda)$ be the number of modes / volume in the range λ & $\lambda + d\lambda$, then density of modes is defined by.

$$dN(\lambda) = \Sigma(\lambda)\, d\lambda \Rightarrow \boxed{\Sigma(\lambda) = \frac{dN(\lambda)}{d\lambda}} \star$$

$$\left\{ dN(\nu) = G(\nu)\, d\nu \right\}^{\star}$$

example:



$n_1, n_2 \equiv$ integers
↓
if we consider
n_1 = constant •
n_2 = constant •

Any Area A, then how many lattice points are inside

The number of squares of unit area inside A

$$\text{no. of points} = \frac{A}{\text{area of unit square}} = A$$

↳ Assume square to be $1 \times 1 = 1$

If we consider the area A

$n >> 1$

$$\frac{\text{Actual no. of points} - \text{Area}}{\text{Actual no. of points}} \approx 0$$

As actual no. of point $\to \infty$

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Volume integral

(+ve) part

only first quarter of sphere

$$N(\lambda) = \frac{4}{3}\pi\left(\frac{2L}{\lambda}\right)^3 \times \left(\frac{1}{8}\right)$$

-ve because radius ↑↑ ⇒ λ↓↓

$$dN(\lambda) = \frac{4\pi}{8}\left(\frac{2L}{\lambda}\right)^2\left(\frac{2L}{\lambda^2}\right)(d\lambda)$$

$$dN(\lambda) = \frac{-4\pi L^3}{\lambda^4} d\lambda$$

as the wave can be polarised along

Polarisation $(n_1 \& n_2)$

Density

$$\frac{dN(\lambda)}{L^3} = \frac{-4\pi}{\lambda^4} d\lambda \times 2$$

$$c = \nu\lambda$$

$$= \frac{8\pi\nu^4}{c^4} \cdot \frac{c \cdot d\nu}{\nu^2}$$

$$= \frac{8\pi\nu^2}{c^3} d\nu = G(\nu)\, d\nu$$

* Radiation is understood as a continuous distribution of amplitude v/s wavelength or amplitude v/s frequency

$$\boxed{G(\nu) = \frac{8\pi\nu^2}{c^3}}$$

total radiation betⁿ ν_1 & ν_2

$$\int_{\nu_1}^{\nu_2} dP(\nu, T) = 8\pi k_B T\frac{(\nu_2^3 - \nu_1^3)}{3c^3}$$

$$= \int \frac{8\pi k_B T(\nu_2^3 - \nu_1^3)}{3c^3}$$

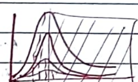
$$\boxed{U(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3} k_B T\, d\nu}$$ * ⟶ Rayleigh - Jean Formula

$$\boxed{\int U(\nu)\, d\nu = \infty}$$

Ultraviolet Catastrophe ⟶ discrepency in classical & experimental nature of graph

↳ this formula is only correct for the right part of the graph

but on the left side the Energy density should increase when λ↑↑



but the formula tells that Energy density ↓↓ over whole graph from left to right

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Planck's Proposal

$$U(\nu)\, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\left(\frac{h\nu}{k_B T}\right)} - 1} d\nu$$, h ⇒ planck's constant

$$h\nu << k_B T$$

Approximation.

$$U(\nu)\, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3\, d\nu}{\frac{h\nu}{k_B T}} = \frac{8\pi}{c^3}\nu^2 k_B T\, d\nu$$

$$h\nu >> k_B T$$

$$\boxed{U(\nu)\, d\nu = \frac{8\pi h}{c^3}\nu^3 e^{\left(-\frac{h\nu}{k_B T}\right)} d\nu}$$ *

$$\boxed{U(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3} k_B T\, d\nu}$$

Proposal :- Modes of oscillations they do not carry continuous energy, rather discrete energy of the form

$$\boxed{E_n = nh\nu}, \quad n = 0, 1, 2, \ldots$$

eg. $E_1 = h\nu$, $E_2 = 2h\nu$, $E_3 = 3h\nu$, ...

∴

$$\boxed{\bar{E} = \frac{\sum_{n=0}^{\infty} nh\nu\, e^{\left(\frac{-nh\nu}{k_B T}\right)}}{\sum_{n=0}^{\infty} e^{\left(\frac{-nh\nu}{k_B T}\right)}}}$$ * Imp

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Endsem Que

Homework :-

$$\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{K_BT}} = \frac{1}{1-e^{-\frac{h\nu}{K_BT}}}$$

$$\sum_{n=0}^{\infty} nh\nu e^{\left(-\frac{nh\nu}{K_BT}\right)} = \frac{h\nu e^{\left(-\frac{h\nu}{K_BT}\right)}}{\left(1-e^{\left(-\frac{h\nu}{K_BT}\right)}\right)^2}$$

Proof LHS from RHS in both expressions

$$dN(\nu) = G(\nu)\,d\nu$$

$$\bar{E} = \frac{h\nu}{\left(e^{\left(\frac{h\nu}{K_BT}\right)}-1\right)}$$ *

$$\left\{ u(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{\left(e^{\left(\frac{h\nu}{K_BT}\right)}-1\right)}\,d\nu \right\}$$ Imp *

Solution: Imp

$$P(n) = \frac{e^{(-E_n/KT)}}{\sum_{n=0}^{\infty} e^{(-E_n/KT)}}$$: single outcome / summation of all posible outcomes

$$\langle E\rangle = \sum_{n=0}^{\infty} E_n P(n) = \frac{\sum_{n=0}^{\infty} E_n e^{(-E_n/KT)}}{\sum_{n=0}^{\infty} e^{(-E_n/KT)}}$$

By plank's Law $\boxed{E = nh\nu}$

$$\therefore \langle E\rangle = \frac{\sum_{n=0}^{\infty} (nh\nu)\, e^{(-E_n/KT)}}{\sum_{n=0}^{\infty} e^{(-E_n/KT)}} = h\nu\,\frac{\sum_{n=0}^{\infty} n e^{(-E_n/KT)}}{\sum_{n=0}^{\infty} e^{(-E_n/KT)}}$$

$$\langle E\rangle = h\nu\,\frac{\sum_{n=0}^{\infty} n x^n}{\sum_{n=0}^{\infty} x^n}$$ where $\boxed{x = e^{-\left(\frac{h\nu}{KT}\right)}}$

$$\frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{\left(e^{\left(\frac{h\nu}{K_BT}\right)}-1\right)}$$

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$$\langle E\rangle = h\nu\,\frac{[1x + 2x^2 + 3x^3 + \dots]}{[1 + x + x^2 + x^3 + \dots]}$$

$$\langle E\rangle = h\nu x\,\frac{[1 + 2x + 3x^2 + 4x^3 + \dots]}{[1 + x + x^2 + x^3 + \dots]}$$

we know from the series expansions

$$\frac{1}{(1-x)^2} = (1 + 2x + 3x^2 + 4x^3 + \dots)$$

$$\frac{1}{(1-x)} = 1 + x + x^2 + \dots$$

$$\langle E\rangle = \frac{h\nu x}{(1-x)} = h\nu\left(\frac{1}{1/x - 1}\right) = \frac{h\nu}{\left(e^{\left(\frac{h\nu}{KT}\right)}-1\right)}$$

$$\boxed{\langle E\rangle = \frac{h\nu}{\left(e^{\left(\frac{h\nu}{KT}\right)}-1\right)}}$$ *

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* Crompton Scattering :-

x - ray = 17.5 kev

wavelength ⟶ 0.07 nm

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$$

m_e → mass of e^-

the wavelength that he observed. | incident wavelength

$$\left\{ \frac{h}{m_e c} = \text{crompton wavelength} \right\}$$

⟶ this cannot be explained by the classical scattering.

Crompton explained the observation by considering elastic scattering of two particles.

In special theory of relativity, a particle of mass "m", energy "$\vec{E}$" and momentum "$\vec{p}$".

$$\boxed{E = \sqrt{p^2c^2 + m^2c^4}} \; *$$

$$\boxed{E = \frac{mc^2}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}}$$

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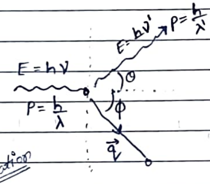
$$\boxed{p = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

If we extend, the relation to a photon which is moving with speed c, unless m = 0

$\Rightarrow$ $\boxed{E = pc}$ $\Rightarrow$ $E = h\nu$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$



we consider the process where the photon hits a free e^- at rest.

Derivation

Energy & momentum conservation :-

$$\left.\begin{aligned} \frac{h}{\lambda} &= \frac{h}{\lambda'}\cos\theta + q\cos\phi \\ \frac{h}{\lambda'}\sin\theta &= q\sin\phi \end{aligned}\right\} \text{momentum conservation}$$

$$\lambda' - \lambda = (1-\cos\theta)$$

$$h\nu + m_e c^2 = h\nu' + \sqrt{q^2c^2 + m_e^2c^4} \quad \Big\} \text{ energy conservation}$$

from conservation of momentum

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 + \frac{h^2}{\lambda'^2}\sin^2\theta = q^2.$$

$$\boxed{\frac{h^2}{\lambda^2} + \frac{h^2}{(\lambda')^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta = q^2}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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$$\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2\right)^2 = q^2 c^2 + m_e^2 c^4$$

minus

$$h^2c^2\left(\frac{1}{\lambda^2} + \frac{1}{(\lambda')^2} - \frac{2}{\lambda\lambda'}\right) + m_e^2 c^4 + 2m_e c^2 hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$h^2c^2\left(\frac{1}{\lambda^2} + \frac{1}{(\lambda')^2} - \frac{2}{\lambda\lambda'}\right) + 2m_e c^3 h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = h^2c^2\left(\frac{1}{\lambda^2} + \frac{1}{(\lambda')^2}\right) - \frac{2h^2c^2\cos\theta}{\lambda\lambda'}$$

$$2m_e c^3 h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{2h^2c^2}{\lambda\lambda'}(1-\cos\theta)$$

$\Delta\lambda = \lambda' - \lambda$, Compton shift

Compton equation →
$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$$

Endsem★ Question based on result

if

1) $\theta = 0$ $\quad \lambda' = \lambda$

2) $\theta = \pi/2$ $\quad \lambda' = \lambda + \frac{h}{m_e c}$

3) $\theta = \pi$ $\quad \lambda' = \lambda + \frac{2h}{m_e c}$

$$\frac{h}{m_e c} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 2.5\ \text{pm (pico meter)}$$

$$= 0.0025\ \text{nm}$$

- nucleus + inner electrons strongly bounded
  ↳ 300 eV
  we have ignored 300 eV ~~17.5 eV~~ in compare to 17.5 KeV

End Sem question

Justify ↳

• Reason :- why do we get two wavelength? Explain. (shift)

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If scattering of inner core, we have to take crompton ~~complete~~ wavelength of whole atom.

★ Planck → Einstein → Crompton

Planck	Einstein	Crompton
Energy of a black body radiation is quantized.	quanta is universal $\boxed{E = h\nu}$	Each packet is like particle with $\boxed{E = h\nu}$ $\boxed{p = \frac{h}{\lambda}}$ ↳ $E = pc$

wave property	particle characteristic
$\omega, \vec{k}$	E, P $\quad E = pc$

→ Wave particle duality
$E = h\nu$
particle characteristic, wave characteristic

- 1924 Louis de Broglie wave / Matter wave — interdependent
  photon is not special & wave characteristic exist also for a massive particle.
  A particle with energy E and momentum P, there is a plane wave with frequency & wavelength

$$\boxed{E = \hbar\omega} \quad ; \quad \boxed{\lambda = \frac{h}{p}}$$

~~$\omega, \vec{k} \leftrightarrow E, \vec{P}$~~

for a particle with energy E and momentum P, is a plane wave associated with wave characteristics given by

$$E = \hbar\omega, \quad \lambda = \frac{h}{p}$$

↳ de Broglie wavelength

wave is called matter wave or de broglie wavelength

OCW-15

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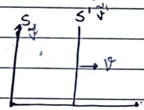
In 1927, Davission-Germer showed experimentally that the electron has wave characteristic consistent with de broglie canjecture.

$$m_e = 9.1 \times 10^{-31}\ \text{kg}$$
$$v = 5 \times 10^{4}\ \text{ms}^{-1}$$
$$\lambda = \frac{h}{mv} = 1.45 \times 10^{-8}\ m = 14.5\ nm.$$

question conceptual Imp

* What is wave ?
* Wave of what ?
* What is waving ?

\# Physical waves are measurable :-



$$\therefore x' = x - vt$$
$$t' = t$$
$$y' = y,\ z' = z$$

$(x, y, z) \longrightarrow S$ (coordinate system for S)
$(x', y', z') \rightarrow S'$ (coordinate system for S')

x' is moving along x axis with speed $\tilde{v}$ in S

$p = m\tilde{v} \rightarrow$ in S

$\tilde{v}'$ in S'

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

in S' $\quad p' = p - mv$ $\quad$ as $\tilde{v}' = \tilde{v} - v$

$$\Rightarrow \lambda' = \frac{h}{p'} = \frac{h}{p - mv} \neq \lambda$$

OCW-16

$\phi \leftarrow$ Galilian Invariant

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this is for a de-broglie wave

$$\lambda' = \frac{h'}{p'} = \frac{h}{p - mv} \neq \lambda$$

$\hookrightarrow$ this is not a characteristic ~~and~~ of physical wave.

Homework prove $\rightarrow$ It is not a physical wave.

. Normal wave analysis for normal wave

Imp Explain question End sem

* Statement : The phase remains invaried (same) from frame to frame.

$$\phi(x, t) = \phi'(x', t')$$

for a physical phase

S $\quad$ S' $\quad$ gallelian invariant

Both S & S' take snapshot of wave at time t.

x_1 & x_2 is distance betⁿ two successive nodes in S
x_1' & x_2' is distance betⁿ two successive nodes in S'

$\frac{\lambda}{2}$ is distance betⁿ two successive nodes.

$$\frac{\lambda}{2} = x_2' - x_1' = (x_2 - vt) - (x_1 - vt) = x_2 - x_1 = \frac{\lambda}{2}$$

$$\frac{\omega'}{k'} = V - v$$

$$\omega' = k'(V - v) = kV\left(1 - \frac{v}{V}\right)$$
$$= \omega\left(1 - \frac{v}{V}\right)$$

$$\phi'(x', t') = k'x' - \omega' t'$$
$$= \phi(x, t)$$

$$\psi'(x', t') = \psi(x, t)$$

wave

E, P, ν $\frac{h}{2\pi} = \hbar$ $2\pi\nu$ $\left(\frac{h}{2\pi}\right)(\nu 2\pi)$

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Homework

$E = p^2/2m$ $E' = \frac{p'^2}{2m}$

Find ~~v~~ some ω' & ω relationship

by using λ & λ' relationship

Also prove $\boxed{\phi'(x',t') \neq \phi(x,t)}$

Also prove using $\boxed{\psi'(x',t') \neq \psi(x,t)}$

Conclusions: Note (1) ψ are not directly measurable in case of de-broglie waves.

(2) debroglie waves ~~are~~ ~~not~~ don't shows Gallelian Invariant.

question # What is the mathematical form of plane wave?

↳ Guess → let's consider it moving along (+ve) x axis:

1) $\sin(kx - \omega t)$
2) $\cos(kx - \omega t)$
3) $e^{i(kx-\omega t)}$
4) $e^{-i(kx-\omega t)}$

* let's try to find wave eqn.

Guess (1) $\nabla^2\psi = \alpha \frac{\partial^2\psi}{\partial t^2}$, here only x is considered

$\therefore \boxed{\frac{\partial^2\psi}{\partial x^2} = \alpha \frac{\partial^2\psi}{\partial t^2}}$

→ this ~~are~~ is solution only if $\omega^2 = \alpha k^2$

$\hbar^2\omega^2 = \alpha\hbar^2 k^2$

$\boxed{E^2 = \alpha p^2}$

but we know clearly E & p satisfy

$\boxed{E = \frac{p^2}{2m}}$

therefore $\frac{\partial^2\psi}{\partial x^2} = \alpha\frac{\partial^2\psi}{\partial t^2}$ does not satisfy

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Guess (2) ~~lets~~ as we know

$E = \frac{p^2}{2m}$

related to ω related to k

$\therefore \frac{\partial\psi}{\partial t} = \alpha\frac{\partial^2\psi}{\partial x^2}$ → only (3) & (4) wave satisfy this

but (1) & (2) does not satisfy

Guess (3)

$\frac{\partial\psi}{\partial t} = -i\omega\psi$

$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$

$-i\omega = \alpha(-k^2)$

$\Rightarrow iE = \alpha\hbar k^2 = \alpha\frac{p^2}{\hbar}$ *

$\frac{ip^2}{2m} = \frac{\alpha p^2}{\hbar}$

* $\Rightarrow \alpha = \frac{i\hbar}{2m}$

$e^{i(kx-\omega t)}$

$\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}$

$\Rightarrow \left\{ i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \right\}$ satisfy.

$\boxed{p = \hbar k}$ & $\boxed{E = \hbar \omega}$

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Homework — Guess (4)

$e^{-i(kx-\omega t)}$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = +\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}} \star$$

equation for (4)th wave

Convention:

plane wave : $e^{i(kx-\omega t)}$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \partial^2 \psi}$$

only this satisfy.

- Wavefunction:

$\Psi(\vec{r},t)$

or $\psi(\vec{r},t)$

Imp — to Read from book — Quantum computers point of view

Assume if ψ_1 & ψ_2 are "allowed" wavefunctions of a quantum system then

$\psi = c_1\psi_1 + c_2\psi_2$ is also 'allowed' — principle of superposition

$c_1 \& c_2 \in \mathbb{C}$

(1) Sine wave is "allowed" wave.

$\psi_1 = \sin(kx - \omega t)$

$\psi_2 = \sin(kx + \omega t)$

$\psi = \sin(kx - \omega t) + \sin(kx + \omega t)$

$\psi = \pm \frac{\pi}{2\omega}, \pm \frac{3\pi}{2\omega}, \ldots$

ICW 20

Axioms of superposition principle.

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$\psi = 0$ at all values of x when $t = \pm\frac{\pi}{2\omega}, \pm\frac{3\pi}{2\omega} \ldots$

$\therefore$ Sine wave is not allowed wave fn.

(2) $\cos(kx - \omega t)$ $\neq$ similarly not allowed

(3) $e^{i(kx-\omega t)}$

$\psi = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}$

$$\star \boxed{\psi = 2\cos kx\, e^{-i(\omega t)}} \star$$

(4) $e^{-i(kx-\omega t)}$

$\|^y$ $\psi = e^{-i(kx-\omega t)} + e^{-i(-kx-\omega t)}$

$$\star \boxed{\psi = 2\cos kx\, e^{i(\omega t)}} \star$$

If both (3) & (4) are allowed,

$\psi = e^{i(kx-\omega t)} + e^{-i(kx-\omega t)} = 2\cos(kx-\omega t)$

$e^{i(kx-\omega t)} \Rightarrow$ de broglie wave for definite ~~manipulation~~ momentum P & Energy E.

Imp — See $\rightarrow$ derive time independent Schrodinger wave eqn.

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}} \rightarrow \text{time dependent free particle Schrodinger eq}^n.$$

2) Statement: If $\psi(\vec{r},t)$ is the wavefunction of particle, then probability of finding the particle in volume ΔV at (x,y,z) at time t is

$$\boxed{P_{\Delta V} = |\psi(\vec{r},t)|^2 \Delta V}$$

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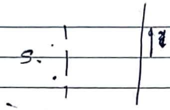
$\Rightarrow$ $|\psi(\vec{r},t)|^2$ = probability density

$|\psi(\vec{r},t)|$ = probability amplitude.

- Reduce the intensity and reduce it's exposure time. we will see dots which are photon



photographic plates

$I \propto |E|^2$ $\quad E = E_0(y)\, e^{i(kx-\omega t)}$

$\sim \cos\left(\frac{\delta}{2}\right)$

you will find the photons only at the places where $|E|^2 \neq 0$ and no photon of $|E|^2 = 0$.

The average effect will be consistent with classical theory.

$|E|^2 \rightarrow$ tells the probability of finding the photon.

- No. of electrons and point y per unit time per unit area $= N(y)$

$$I(y) = N(y)\,\frac{p^2}{2m}$$

Quantum Mechanics tells that

$$I(y) \propto |\psi|^2$$

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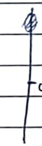
$$N(y) \propto |\psi|^2$$

If N_{total} is the no. of electrons emitting per unit time then the probability of finding the electron at y in area ΔA is

$$\underset{\text{in } \Delta A/t}{\text{Probability}} = \frac{N(y)\,\Delta A}{N_{Tot}}$$



= probability of finding electron in volume ΔV.

$$P_{\Delta V} = \frac{N(y)}{N_{tot}}\,\Delta A\,\Delta t = \frac{N(y)\,\Delta V}{N_{tot}\,v_x}$$

volume

velocity

$P_{\Delta V} = |\psi|^2\,\Delta V$

$\longrightarrow$ Probability density.

$p^2 = p'$

$E = \frac{p^2}{2m} \quad \frac{h^2}{} \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial \phi}{\partial t}$ Date: __/__/__

$e^{i(kx - \omega t)}$

3 sol^n

$\phi'(x', t') \neq \phi(x, t)$

$x' = x - vt$ $\qquad \psi_1 = e^{i(kx - \omega t)}$

$\tilde{v}' = \tilde{v} - v$ $\qquad \psi_1 = e^{i(\frac{2\pi}{\lambda} x - \omega t)}$

$p = \frac{h}{\lambda}$

$\psi_2 = e^{i(\frac{2\pi}{\lambda'} x' - \omega' t')}$

$p' = \frac{h}{\lambda'}$ $\qquad p = mv$

$mv = \frac{h}{\lambda}$ $\qquad E = p/c = \frac{hc}{\lambda \times c}$ $\qquad p = E/c$

$\tilde{v} = \frac{h}{m\lambda}$ $\qquad E = pc = \frac{hc}{\lambda}$

$\tilde{v}' = \frac{h}{m\lambda'}$

$\tilde{v}' - \tilde{v} = \frac{h}{m}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$\frac{m(\tilde{v}' - \tilde{v}')}{h} + \frac{1}{\lambda'} = \frac{1}{\lambda}$

$\frac{2\pi m(\tilde{v}' - \tilde{v})}{h} + \frac{2\pi}{\lambda'} = \frac{2\pi}{\lambda}$

$x' = x - vt$

~~Multiply by x~~

$\frac{2\pi}{\lambda'} = \frac{2\pi}{\lambda} - \frac{2\pi}{h} m(\tilde{v}' - v)$

Multiply by x'

$\frac{2\pi}{\lambda'} x' = \frac{2\pi}{\lambda}(x - vt) - \frac{2\pi}{h} m(\tilde{v}' - v)(x - vt)$

$\frac{2\pi x'}{\lambda'} = \left\{\frac{2\pi}{\lambda} - \frac{2\pi}{h} m(\tilde{v}' - \tilde{v})\right\} x + \left(\frac{2\pi}{\lambda} - \frac{2\pi}{h} m(\tilde{v}' - v)\right)(-vt)$

... eq^n (1)

$x' = x - vt$

$x = x' + vt$ $\qquad$ Date: __/__/__

$e^{i(kx - \omega t)}$

$e^{i\left(\frac{2\pi}{\lambda}(x' + vt) - \frac{2\pi}{\lambda} V t\right)}$

$e^{i\left(\frac{2\pi}{\lambda}(x') - \frac{2\pi}{\lambda}(V - v)t\right)}$

$- \frac{2\pi V}{\lambda}(1 - v/V) t$

$\frac{2\pi}{\lambda'}(x')$

$\omega' = \omega(1 - v/V) \qquad \frac{V - v}{V}$

$\lambda' = \lambda$

$\omega' = \tilde{v}' k = \frac{\tilde{v}' 2\pi}{\lambda'}$ $\qquad \psi_2 = e^{i(k' x' - \omega' t)}$

$\omega' t = \left(\frac{\tilde{v}' 2\pi}{\lambda'} t\right)$ ... eq^n (2)

$e^{i(kx - \omega t)}$ $\qquad - \frac{2\pi}{\lambda} \tilde{v}' t$

$\boxed{\frac{\omega'}{2\pi \tilde{v}'} = \frac{1}{\lambda'}}$

$\frac{2\pi}{\lambda}(x - vt)$

$\boxed{\frac{2\pi}{\lambda} x}$

$\frac{2\pi}{\lambda'} x' = \boxed{\frac{2\pi}{\lambda'} x} - \frac{2\pi}{\lambda'} vt$

$\frac{2\pi}{\lambda} \cdot k, x$

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★ The probability of finding the particle in vol ΔV at (x,y,z) at time t. is

$$P_{\Delta V} = C|\psi|^2 \Delta V$$

$$\psi' = A\psi \qquad \int |\psi|^2 dV = 1$$

constant   ↳ normalisation.

$$\int P_{\Delta V} = 1 = C\int |\psi|^2 dV = \frac{C}{A^2}\int |\psi'|^2 dV$$

$$\boxed{A = \sqrt{C}} \star$$

$$\left\{ P_{\Delta V} = |\psi|^2 \Delta V \right\}$$

★ A particle of definite momentum p & Energy E.

$$\psi = Ae^{i(kx-\omega t)}$$

$$p = \hbar k \quad , \quad E = \hbar\omega$$

Strange Results;

$$|\psi|^2 = A^2$$

$$\int_{-\infty}^{\infty} dx\, |\psi|^2 = \infty$$

Phase velocity

$$V_p = \frac{\omega}{k} = \frac{E}{p} = \frac{p^{\cancel{2}}}{2m/\cancel{p}} = \frac{p}{2m} = \frac{v}{2}$$

speed of particle

$$\left\{ V_p = \frac{v}{2} \right\}$$

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★

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$e^{i(k_1x-\omega_1 t)}, \; e^{i(k_2x-\omega_2 t)}$$

$$\psi = C_1 e^{i(k_1x-\omega_1 t)} + C_2 e^{i(k_2x-\omega_2 t)}$$

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \phi(k)\, e^{i(kx-\omega(k)t)}$$

wavepacket

$\phi(k)$ is the amplitude of wave depending upon k.

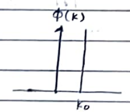
$$\psi(x,0) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \phi(k) e^{ikx}$$

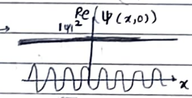
1) $\phi(k) = A\delta(k-k_0)$

$$\psi(x,0) = \frac{A}{\sqrt{2\pi}} e^{ik_0x}$$

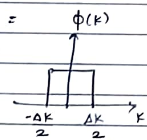
dirac-delta function.

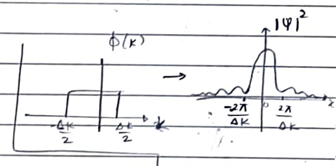
at $k_0 \to (\phi(k_0) \to \infty)$





② $\phi(k) =$

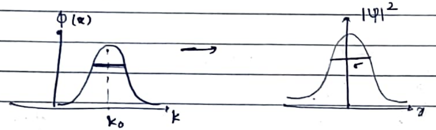




$$\psi(x,0) = \frac{1}{\sqrt{2\pi}}\int_{-\Delta k/2}^{\Delta k/2} dk\, e^{ikx} = \frac{A\sin\left(\frac{\Delta k x}{2}\right)}{\left(\frac{\Delta k x}{2}\right)}$$

3) $\phi(k) = A e^{-\frac{(k-k_0)^2 \sigma^2}{2}}$

$$\psi(x,0) = \frac{A}{\sigma} e^{-\frac{x^2}{2\sigma^2}} e^{ik_0 x}$$



Our inability to find the wavefunction that gives precise value of position and momentum of the particle.

→ Heisenberg uncertainity principle.

*Heisenberg uncertainty principle

• Data N points $x_i \to i = 1, 2, \dots N$

mean value

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \to \bar{x} = \int_{-\infty}^{\infty} |\psi|^2 x \, dx$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$\sigma^2 = \overline{(x_i^2)} - \bar{x}^2$$

$$\therefore \sigma^2 = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx - \left(\int_{-\infty}^{\infty} x |\psi|^2 dx\right)^2$$

$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n p(x) \, dx = \int_{-\infty}^{\infty} x^n |\psi|^2 dx$

$\int_{-\infty}^{\infty} p(x) dx = \int_{-\infty}^{\infty} |\psi|^2 dx$

σ ⇒ represents uncertainity in the position of the particle.

$$\boxed{\sigma_x \sigma_p \geq \frac{1}{2} \underbrace{|\langle [\hat{x}, \hat{p}_x] \rangle|}_{h/2}}$$

Heisenberg uncertainity principle.

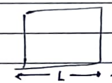
Δx = uncertainity / variance in the position of momentum of particle

It is not possible by any measurement process to determine & momentum of the particle with an accuracy greater than

$$\Delta x \Delta p = \frac{\hbar}{2}$$

for all process $\Delta x \Delta p \geq \frac{\hbar}{2}$ ← acts as a bound.

• eg. Consider a free particle in a box



$\Delta x_{max} = L$ for this box

$$E = \frac{p^2}{2m}$$

according to classical mechanics → $E_{min} = 0$

In quantum mechanics

$$\bar{E} = \frac{\overline{p^2}}{2m}$$

$\bar{P} = 0$

mean momentum = 0

$$(\Delta P)^2 = \overline{p^2}$$

we know

$$\Delta x \Delta p > \frac{\hbar}{2}$$

$$\Delta P_{min} > \frac{\hbar}{2 \Delta x_{max}}$$

$$\Delta P > \frac{\hbar}{2L}$$

$$\overline{\Delta P^2} > \frac{\hbar^2}{4L^2}$$

$$\bar{E} > \frac{\hbar^2}{4L^2(2m)}$$

$$\bar{E} > \frac{\hbar^2}{8mL^2}$$

consequence of uncertainity principle.

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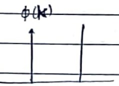
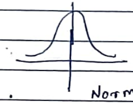
$$\bar{E} = \frac{\bar{p}^2}{2m} \gtrless \frac{(\Delta P)^2}{2m}$$

$\bar{E}_1, \bar{E}_2, \bar{E}_3 \ldots$ are results of various experiment & we want to calculate $\bar{E}$

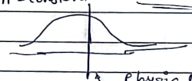
$$\bar{E} = \frac{1}{N}\sum_{i=1} E_i$$

$E_i \neq 0$





Normalization



physical wave

1) de broglie wave cannot physically be associated to a particle.

2) Box - normalisation.

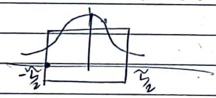
$$\left\{ \int_{-\tilde{L}/2}^{\tilde{L}/2} |\psi|^2 dx = 1 \right\}$$

we assume

$\psi = Ae^{i(kx-\omega t)}$

$$\boxed{A = \frac{1}{\sqrt{\tilde{L}}}}$$

$\tilde{L}$ ← this is the small length of element assumed



~~One dim Box~~

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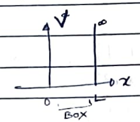
* One dimensional Box



Potential ~~Energy~~ ✓ $V(x) = \infty$ for $x < 0$ & $x > L$

$= 0$ for $0 < x < L$

we use word potential here but it means potential energy (use same) $V(x)$ for both.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}, \quad E = \frac{p^2}{2m}$$

$$E = \frac{p^2}{2m} + V(x,t)$$

* Proposal:

$$\boxed{i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\,\psi(x,t).}$$

*

→ this can be solved using seperation of variable → find in the tutorial session ① notes.

$$\psi(x,t) = \tilde{\psi}(x)\, f(t)$$

$$\underbrace{i\hbar \frac{\partial f}{\partial t}\frac{1}{f} = E \; ; \; \frac{-\hbar^2}{2m}\frac{\partial^2 \tilde{\psi}}{\partial x^2} = E\tilde{\psi}}_{}$$

these can be obtained from

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \Rightarrow i\hbar\, \tilde{\psi}(t) \frac{\partial f(t)}{\partial t} = -\frac{\hbar^2}{2m} f(t) \frac{\partial^2 \tilde{\psi}}{\partial x^2}$$

constant as consider this E

function of (x) & on right side, on the left function of (t)

$$\frac{i\hbar}{f(t)}\frac{\partial f(t)}{\partial t} = \left(\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)\frac{1}{\tilde{\psi}}$$

solution

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this is the solution obtained from (1) + (11)

$$\boxed{f(t) = e^{-i\frac{Et}{\hbar}}}$$ if potential is independent of t

Important eqn(1)
$$\boxed{\frac{-\hbar^2}{2m}\frac{d^2\psi}{\partial x^2} + V(x)\tilde{\psi}(x) = E\tilde{\psi}(x)}$$

These two eqn's are known as time independent

eqn(2)
$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2\tilde{\psi}}{\partial x^2} = E\tilde{\psi}}$$

from last page.

$$\Rightarrow \quad \psi(x,t) = \tilde{\psi}(x)\, e^{\left(\frac{-iEt}{\hbar}\right)}$$

E = energy of the particle is ψ.

$$\frac{-\hbar^2}{2m}\frac{d^2\tilde{\psi}}{dx^2} + V(x)\tilde{\psi}(x) = E\tilde{\psi}(x)$$

$\tilde{\psi}(x) = 0$ for $x < 0$ & $x > L$

$$P(x,t) = |\psi(x,t)|^2 = |\tilde{\psi}(x)|^2$$

Physical requirement probability density should be well defined at all values of x

$\Rightarrow$ $\tilde{\psi}$ should be continuous at all x.

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for $0 < x < L$

$$E\tilde{\psi} = \frac{-\hbar^2}{2m}\frac{\partial^2\tilde{\psi}}{x^2}, \quad E = \frac{p^2}{2m}$$

$E > 0$

$$\frac{d^2\tilde{\psi}}{dx^2} + k^2\tilde{\psi} = 0$$

$$\boxed{k = \sqrt{\frac{2mE}{\hbar^2}}}$$

$$\tilde{\psi}(x) = A\cos kx + B\sin kx$$

$\psi(0) = 0 \Rightarrow A = 0$

$\tilde{\psi}(L) = 0 \rightarrow B\sin(kL) = 0$

$\Rightarrow kL = \pm\pi, \pm 2\pi, \pm 3\pi \dots = n\pi$

$$\tilde{\psi} = B\sin\left(\frac{n\pi x}{L}\right)$$

$n = 1, 2, 3 \dots$

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2\pi^2}{2m}$$

find ~~Ba~~ normalisation constant

Homework

Imp.

$$\boxed{\psi_n(x,t) = B\, e^{\left(\frac{-iE_n t}{\hbar}\right)}\sin\left(\frac{n\pi x}{L}\right)}$$

$n = 1, 2, 3 \dots$

$$\psi_n(x,t) = \begin{cases} 0 & x < 0 \\ B e^{\left(\frac{-iE_n t}{\hbar}\right)}\sin\left(\frac{n\pi x}{L}\right) & 0 < x < L \\ 0 & x > L \end{cases}$$

$$\boxed{\int_{-\infty}^{\infty} |\psi_n(x,t)|^2 dx = 1}$$ $\Longrightarrow$ find the value of B from this relation.

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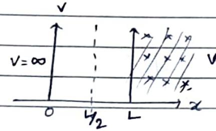
* Infinite well potential / particle in a box



potential is same about ← L/2 →

$$V(x) = \begin{cases} \infty & x<0,\ x>L \\ 0 & 0<x<L \end{cases}$$

(*) time independent schrodinger eqn

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi(x) = E\,\psi(x)$$

$$\psi(0) = \psi(L) = 0$$

$$\psi_n(x) = d_n \sin\left(\frac{n\pi x}{L}\right) \quad , n = 1, 2, 3 \ldots$$

d_n normalization constant

$$\boxed{E_n = \frac{n^2 \hbar^2 \pi}{2mL^2}}$$

$$\int_0^L |\psi_n(x)|^2\, dx = \cancel{0}\ 1$$

$$d_n^2 \int_0^L \sin^2 \frac{n\pi x}{L}\, dx = 1$$

$$\frac{d_n^2}{2} \int_0^L \left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx = 1$$

$$\left\{ d_n = \sqrt{\frac{2}{L}} \right\}$$

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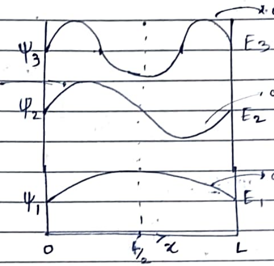
$$\star \quad \boxed{\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$



$\psi_n(x)$ has definite parity under $x \to L - x$

odd/even

$$\psi_n(x) = (-1)^{n-1}\, \psi_n(L-x)$$

All the waveforms $\{\psi_n(x)\}$, form orthonormal basis with respect to the Hermition inner product

$$(\psi_1, \psi_2) = \int_0^L \psi_1^*(x)\, \psi_2(x)\, dx \quad \star$$

$(\psi_1, \psi_2)^* = (\psi_1, \psi_2)$

$$(\psi_1, \psi_2)^* = \int_0^L \psi_1(x)\, \psi_2^*(x)\, dx$$

complex conjugation

* Property ① $(\psi_1, \psi_2) = \overline{(\psi_2, \psi_1)}$

② $(\psi, \psi) \geq 0$

③ $(\psi, \psi) = 0 \Rightarrow \psi = 0$

$$(\psi_n, \psi_m) = \frac{2}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx$$

$$= \frac{1}{L}\int_0^L \left(\cos\frac{(n-m)\pi x}{L} - \cos\frac{(n+m)\pi x}{L}\right) dx$$

n & m are +ve integers

$$= \frac{1}{L}\int_0^L \cos\frac{(n-m)\pi x}{L}\, dx - \left[\sin\frac{(n+m)\pi x}{L}\right]_0^L$$

at $n = m$

$$\frac{1}{L}\int_0^L dx = 1 = \delta_{nm}$$

IMP

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Any function $f(x)$ satisfying $f(0) = f(L) = 0$

fourier series solution

$$f(x) = \sum_{n=1}^{\infty} C_n \psi_n(x)$$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

Most general solution of schrodinger eq^n

$$\psi(0) = \psi(L) = 0$$

$$\psi(x,t) = \sum_{n=1}^{\infty} C_n \psi_n e^{-\left(\frac{iE_n t}{\hbar}\right)}$$

consider $C_n \in \mathbb{C}$

any complex constant.

$$\frac{\hbar \partial \psi(x,t)}{dt} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

↓ constants.

What are C_n's ?

$$\int_0 |\psi(x,t)|^2 dx = \left(\sum_{n=1}^{\infty} C_n \psi_n(x), \sum_{n=1}^{\infty} C_m \psi_m(x)\right)$$

$$= \sum_{n=1}^{\infty} C_n^* C_m (\psi_n, \psi_m)$$

how much ψ_n we have in ~~state~~ ϕ

this is given by $|C_n|$

$$1 = \sum_{n=1}^{\infty} C_n^* C_m \delta_{n,m}$$

$$1 = \sum_{n=1}^{\infty} |C_n|^2$$

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Endsem

Q) Explain physical significance of $|C_n|$.

$|C_n|^2$ = probability of finding energy state

ψ_n in ψ

$$\bar{E} = \sum_{n=1}^{\infty} |C_n|^2 E_n \quad \text{where} \quad \sum_{n=1}^{\infty} |C_n|^2 = 1$$

Average energy

probability for n^{th} Energy state.

energy for n^{th} state

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \quad 0 < x < L$$

$$= 0, \quad \text{elsewhere}$$

$$\psi(x,0) = A\psi_1(x,0) + B\psi_2(x,0)$$

$$\bar{E} = \frac{E_1|A|^2 + E_2|B|^2}{|A|^2 + |B|^2}$$

$$|\psi_n(x,t)| = \left|\psi_n(x,0)\, e^{-\left(\frac{iE_0 t}{\hbar}\right)}\right| = |\psi_n(x,0)|$$

$$\bar{x} = \int_0^L x|\psi_n(x,t)|^2 dx$$

$$= \frac{2}{L}\int_0^L x \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L}\int_0^L x\left[1 - \cos\frac{2n\pi x}{L}\right] dx$$

Last ③ Lectures
important

Date:___/___/___

$$= \frac{1}{L}\left\{\frac{L^2}{2} - \cancel{\int_0^L x\cos\left(\frac{2n\pi x}{L}\right)dx}^{\,0}\right\}$$

$$\star\ \boxed{\bar{x} = \frac{L}{2}}$$

~~$\Psi(x,t) =$~~

$$\Psi(x,0) = \sum_{n=1}^{\infty} C_n \Psi_n(x,0)$$

$$\sum_{n=1}^{\infty} |C_n|^2 = 1$$

$$\Rightarrow \bar{x} = \int_0^L x\,|\Psi(x,0)|^2\,dx$$

$$= \int_0^L x \sum_{\substack{n=1\\m=1}}^{\infty} C_n^* C_m \Psi_n^* \Psi_m\,dx$$

Homework

$$\boxed{\bar{x} = \sum_{n,m} C_n^* C_m \int_0^1 x\,\Psi_n^* \Psi_m\,dx}\ \star$$

4 terms

n = m
n ≠ m

average position (already calculated)

need to be calculated.

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$$\bar{x} = \int_0^L x|\psi|^2 dx$$

$$\psi = \sum_{n=1}^{\infty} c_n \psi_n$$

$$\bar{E} = \sum_{n=1}^{\infty} E_n |c_n|^2$$

Any linear combination of ψ is a solution.

$$\psi = \psi_n$$

$$\bar{E} = E_n$$

(*) $\psi \rightarrow$ some wavefunction

(*) How do we extract momentum / Energy.

$\Rightarrow$

For a definite momentum $p = \hbar k$

$$\left.\begin{array}{l}\psi = e^{i(kx - \omega_k t)}, \quad \bar{E} = \hbar\omega_k = \dfrac{p^2}{2m} \\ \psi = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} dk\, \phi(k) e^{i(kx - \omega_k t)}\end{array}\right\}$$

Some operation hitting on ψ gives momentum.

operator:

momentum :-

$$\hat{p}\,\psi_{dB} = p\psi$$

we put cap to assume it as quantum mechanical operator.

$\psi_{dB} \Rightarrow$ de-Broglie wavefunction.

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\hat{p}\cdot\psi_{dB} = \frac{\hbar}{i}(ik)\,\psi_{dB} = \hbar k\,\psi_{dB}$$

$$\hat{p}\cdot\psi_{dB} = p\,\psi_{dB}$$

$p = \hbar k$

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$$\int_{-\infty}^{\infty} |\psi|^2 dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2\, \phi^*(k_1)\phi(k_2)\, e^{i(k_2-k_1)x}\, e^{-i(\omega_{k_2}-\omega_{k_1})t}$$

Note: last term is $e^{-i((\omega)_{k_2}-(\omega)_{k_1})t}$

$$\left\{ \int_{-\infty}^{\infty} dx\, e^{i(k_2-k_1)x} = 2\pi\,\delta(k_2-k_1) \right\}$$

dirac delta function

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-\infty}^{+\infty} dk_1 \int_{-\infty}^{\infty} dk_2\, \phi^*(k_1)\phi(k_2)\,\delta(k_2-k_1)\, e^{-i(\omega_{k_2}-\omega_{k_1})t}$$

$$= \int_{-\infty}^{\infty} dk_1 |\phi(k_1)|^2$$

$\longrightarrow$ probability density having the momentum k_1.

$$\bar{p} = \hbar\bar{k} = \hbar\int_{-\infty}^{\infty} dk\, |\phi(k)|^2 k$$

$$\bar{p} = (\underset{①}{\psi}, \underset{②}{\hat{p}\psi})$$

$\left\{ \hat{p} = \dfrac{\hbar}{i}\dfrac{\partial}{\partial x} \right\}$ operator.

$$= \int_{-\infty}^{\infty} dx\, \underbrace{\psi^*}_{①}\, \underbrace{\frac{\hbar}{i}\frac{\partial \psi(x)}{\partial x}}_{②}$$

$$\bar{p} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dk_1\, \phi^*(k_1)\phi(k_2)(\hbar k_2)\, e^{i(k_2-k_1)x}\, e^{-i(\omega_{k_2}-\omega_{k_1})t}$$

integrate over x

$$= \int_{-\infty}^{\infty} dk_1 dk_2\, \phi^*(k_1)\phi(k_2)(\hbar k_2)\,\delta(k_2-k_1)\, e^{i(\omega_{k_2}-\omega_{k_1})t}$$

$$= \int_{-\infty}^{\infty} dk_1 (\hbar k_1) |\phi(k_1)|^2$$

★ ~~#~~ ψ and operator $\hat{O}$
then the average

$$\bar{O} = (\psi, \hat{O}\psi)$$
$$= \int_{-\infty}^{\infty} dx\, \psi^* \hat{O} \psi$$

Imp endsem que.

- position operator

$$\bar{x} = \int_{-\infty}^{\infty} x\, \psi^* \psi\, dx = \int_{-\infty}^{\infty} \psi^* (\hat{x}\, \psi(x))\, dx$$

$$\boxed{\hat{x} f(x) = x f(x)}$$ ★

- operator.

momentum

$$\hat{p}\psi_{dB} = p\psi$$
$$\boxed{\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}}$$ ★

$$\hat{p}\psi_{dB} = \frac{\hbar}{i}(ik)\psi_{dB} = \hbar k\, \psi_{dB}$$
$$= p\,\psi_{dB}$$

$$E = \frac{p^2}{2m}$$
$$\hat{E} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

$$\psi = \sum C_n \psi_n$$
$$\bar{E} = \sum_n |C_n|^2 E_n$$

- Energy operator / Hamiltonian :-

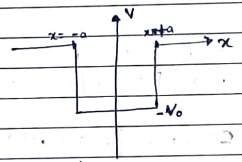
$$\hat{E} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = -\frac{h^2}{2m}\frac{\partial^2}{\partial x^2} + V(\hat{x})$$ ★ Energy operator

$$\bar{E} = (\psi, \hat{E}\psi) = \int_{-\infty}^{\infty} dx\, \psi^* (\hat{E}\psi(x))$$

wherever x appears replace it by $\hat{x}$

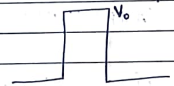
$$\boxed{\hat{E}\psi(x) = -\frac{h^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x)}$$

Potential well (Boundary)



$$V(x) = \begin{cases} -V_0, & -a < x < a \\ 0, & \text{elsewhere} \end{cases}$$

~~$V_0 > 0$~~ $V_0 > 0$



$$= \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

I $\quad -\frac{h^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x) \Rightarrow x < -a$

II $\quad -\frac{h^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi_x = -E\psi(x) \Rightarrow -a < x < a$

III $\quad -\frac{h^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x) \rightarrow x > a.$

→ $\psi(x)$ should be continuous.

Two scenario $E<0$, $E>0$

~~$E<0$~~

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(I) $\dfrac{d^2\psi}{dx^2} = k^2\psi$, $\left\{ k^2 = \dfrac{2m|E|}{\hbar^2} \right.$

$\psi(x) = Ae^{kx} + Be^{-kx}$

(II) $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} = (V_0 - |E|)\psi(x)$, $\left\{ l = \sqrt{\dfrac{2m}{\hbar}(V_0 - |E|)} \right.$

$\psi(x) = C\cos lx + D\sin lx$

(III) $\psi(x) = Ee^{kx} + Fe^{-kx}$

$\because x = -a$

(1) $Ae^{-ka} = C\cos la - D\sin la$

(2) $Ake^{-ka} = l[C\sin la + D\cos la]$

(3) $C\cos la + D\sin la = Fe^{-ka}$

(4) $l[-C\sin la + D\cos la] = -kFe^{-ka}$

Add

(1) + (3) $\longrightarrow$ $2C\cos la = (A+F)e^{-ka}$

(2) − (4) $\longrightarrow$ $2lC\sin la = (A+F)ke^{-ka}$

Homework

evaluate the constants by normalization $\int|\psi|^2 dx = 1$

Divide both. eqn's given above.

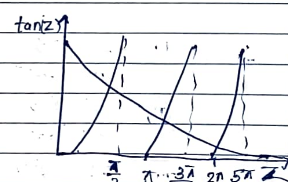
$l\tan(la) = k \Rightarrow \tan\left(a\sqrt{\frac{2m}{\hbar^2}(V_0-|E|)}\right) = \sqrt{\dfrac{|E|}{V_0-|E|}}$

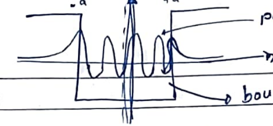
Homework

$z = al$

$z_0 = \dfrac{a}{\hbar}\sqrt{2mV_0}$

$\left\{ \tan z = \sqrt{\left(\dfrac{z_0}{z}\right)^2 - 1} \right.$





probability density of finding ψ in $-a$ to a region is maxm.

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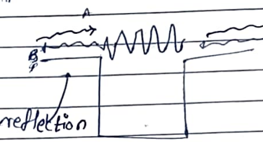
Homework

Solve for the constant A, B, C, D, E by using continuity equations & derivative ~~as~~ determine as a function of V_0 & E



$\psi(x) = Ae^{ikx} + Be^{-ikx}$

Assume them as waves.

$\psi(x) = Fe^{ikx} + Fe^{-ikx}$ $\to 0$

we consider it as transmission wave.

$r = \dfrac{|B|^2}{|A|^2}$

function of $f(V_0, E, a)$

$T = \dfrac{|E|^2}{|A|^2}$

function $\tilde{f}(V_0, E, a)$